

Unit 9 Lesson 4: Compound Probability (Addition Rule and Mutually Exclusive)

I. Review of Basic Probability and Notation

Probability is a numerical value between 0 and 1 that indicates the likelihood of an event or events occurring.

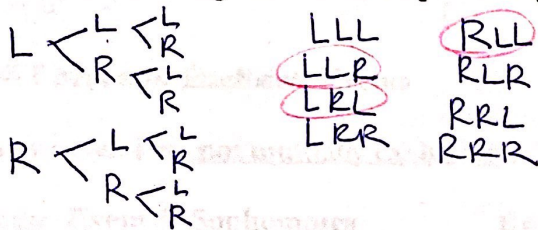
Prior to calculating the probability, it is important to create a sample space to identify all of the possible outcomes and what outcomes would be considered a success. Venn Diagrams, tree diagrams, lists, and frequency tables are useful.

Notation

$P(A \cap B)$ Intersection \Rightarrow AND \Rightarrow what ^{elements} do sets A and B share in common

$P(A \cup B)$ Union \Rightarrow OR \Rightarrow all elements of set A put together with all elements of set B.

Example 1: You are driving in your driver's ed class. The instructor tells you that you must make three turns on your route. Use the tree diagram to find the probability of making exactly two left turns, or $P(\text{exactly two left turns})$



$$P(\text{exactly 2 left turns}) = \frac{3}{8}$$

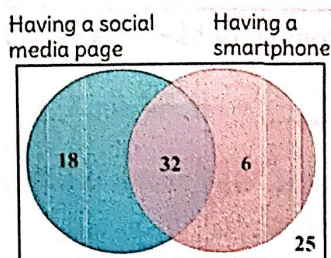
Example 2: Use the frequency table below to find the following probabilities.

	Jrs	Srs	Total
Agree	12	11	23
Disagree	8	15	23
Total	20	26	46

a. $P(\text{Agree}) = \frac{23}{46} = \frac{1}{2}$

b. $P(\text{Jrs}) = \frac{20}{46} = \frac{10}{23}$

Example 3: Use the Venn Diagram below to find the probabilities.



a. $P(\text{social media page}) = \frac{50}{81}$

b. $P(\text{neither social media nor smartphone}) = \frac{25}{81}$

In example 3, there are two events. If we wanted to determine the probability of having a social media page OR having a smartphone, we need to consider if the events could occur at the same time. Can an individual have a social media page AND a smartphone?

YES! Shown in the middle.

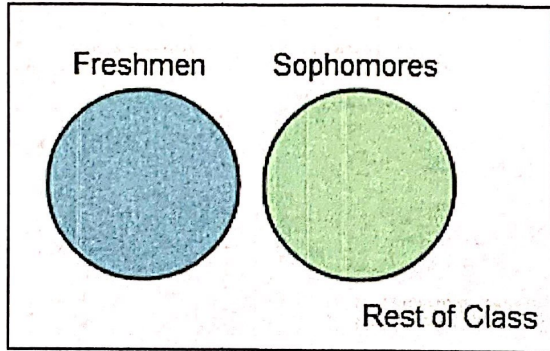
When events occur simultaneously we have to be careful counting the events as we don't want to over count the number of successful outcomes or the total number of outcomes by counting the individuals that have both twice.

II. Mutually Exclusive Events:

When two events are mutually exclusive it is impossible for the events to happen at the same time.

Example: Event A: Freshman

Event B: Sophomore



Suppose you select one student at random from the class.

One person cannot simultaneously be Freshman and Sophomore!

Therefore, $P(\text{Freshman} \overset{\text{and}}{\cap} \text{Sophomore}) = \underline{0}$

For all mutually exclusive events, $P(A \cap B) = \underline{0}$

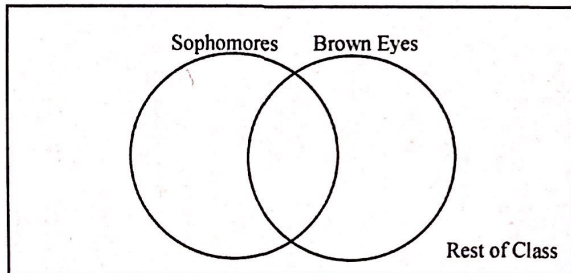
These are also sometimes called disjoint events because the two circles are disjointed or not joined.

II. NOT Mutually Exclusive Events

When two events are not mutually exclusive, it is possible for the events to happen simultaneously.

Example: Event A: Sophomores

Event B: Brown Eyes



Suppose you select one student at random from the class.

What is $P(\text{Sophomore or Brown Eyes})$?

If we simply add sophomores and the individuals with brown eyes...

 Sophomores + Brown Eyes = individuals, then $P(\text{Sophomore or Brown Eyes})$ would be

Is this correct? NO because we overcounted the # of students. We counted the brown-eyed sophomores twice.

Since it is possible for one student to simultaneously be a sophomore and have brown eyes, we need to be sure that we do not count the students that are BOTH or OVERLAPPING twice!

We need to subtract the students that overlap in both categories since they were counted twice.

Since $P(\text{Sophomore} \cap \text{Brown Eyes}) = \underline{\hspace{2cm}}$

We can find $P(\text{Sophomore or Brown Eyes})$...

Addition Rule:

For any two Simultaneous events, A or B,

$$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

- Remember that if two events are mutually exclusive, $P(A \cap B) = 0$

Examples:

- You randomly choose one of the integers 1 – 10. What is the probability of choosing either an odd number or an even number?
 - Calculate the probability. $1, 3, 5, 7, 9$ $2, 4, 6, 8, 10$ none
 $P(\text{odd or even}) = P(\text{odd}) + P(\text{even}) - P(\text{both})$
 $5/10 + 5/10 - 0/10 = 10/10 = 1$
 - Are these events mutually exclusive or not mutually exclusive? Why?
 $ME \Rightarrow$ #s cannot be odd and even at the same time.
- You randomly choose one of the integers 1 – 10. What is the probability of choosing a number less than 5 or odd?
 - Calculate the probability. $1, 2, 3, 4$ $1, 3, 5, 7, 9$ $1, 3$
 $P(\# < 5 \text{ or odd}) = P(\# < 5) + P(\text{odd}) - P(\text{both})$
 $4/10 + 5/10 - 2/10 = 7/10$
 - Are these events mutually exclusive or not mutually exclusive? Why?
 $NME \Rightarrow$ There are #s less than 5 that are also odd.
- A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the first 10 letters of the alphabet on it or with a vowel on it?
 - Calculate the probability. $a, b, c, d, e, f, g, h, i, j$ a, c, e, i, o, u a, c, e, i
 $P(\text{1st 10 letters or vowel}) = P(\text{1st 10}) + P(\text{vowel}) - P(\text{both})$
 $10/26 + 5/26 - 3/26 = 12/26 = 6/13$
 - Are these events mutually exclusive or not mutually exclusive? Why?
 $NME \Rightarrow$ There are vowels in the 1st 10 letters of the alphabet.
- You are rolling a fair six-sided die. What is the probability that you roll an...
 - Odd number or a 2
 $P(\text{odd}) + P(2) - P(\text{both})$
 $3/6 + 1/6 - 0/6 = 4/6 = 2/3$
 - Even number or number less than 3
 $P(\text{even}) + P(< 3) - P(\text{both})$
 $3/6 + 2/6 - 1/6 = 4/6 = 2/3$
 - Odd number or number greater than 3
 $P(\text{odd}) + P(> 3) - P(\text{both})$
 $3/6 + 3/6 - 1/6 = 5/6$
 - Circle the Mutually Exclusive events for a – d.

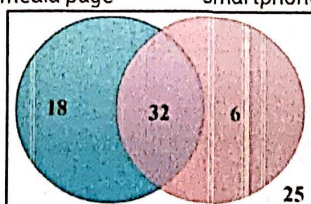
5. Use the frequency table to determine the following probabilities.

	Jrs	Srs	Total
Agree	12	11	23
Disagree	8	15	23
Total	20	26	46

- $P(Jr \cup Agree) = P(Jr) + P(Agree) - P(\text{both})$
 $20/46 + 23/46 - 12/46 = 31/46$
- $P(Sr \cup Disagree) = P(Sr) + P(Disagree) - P(\text{both})$
 $26/46 + 23/46 - 15/46 = 34/46 = 17/23$

6. Use the Venn Diagram to determine the following probabilities.

Having a social media page Having a smartphone



- $P(\text{Have social media page or have a smartphone})$
 $18/81 + 38/81 - 32/81 = 56/81$
- $P(\text{Have a smart phone or having neither})$
 $38/81 + 25/81 = 63/81 = 7/9$