Unit 8 Part 2 Lesson 6: Simplifying and Solving with Rational Exponents

I. Product Rule

Rule: $(a^m)(a^n) =$ _____

Examples:

- 1. $(x^3)(x^5)$ 2. $(m^4)(m)$ 3. $(2y^2)(-4y^5)$
- 4. $(x^{\frac{1}{2}})(x^{\frac{3}{2}})$ 5. $(7^{\frac{3}{4}})(7^{\frac{1}{4}})$ 6. $(5m^{\frac{1}{3}})(-m^{\frac{4}{3}})$ 7. $(\sqrt{x})(\sqrt[2]{x^3})$

II. Power Rules

Rules: $(a^m)^n =$	$(ab)^m = $	$- \qquad \left(\frac{a}{b}\right)^m = -$	
Examples:			
8. $(x^2)^3$	9. $(2x^2y)^3$	10. $\left(\frac{x}{y}\right)^5$	11. $\left(\frac{x}{3y^2}\right)^4$
12. $\left(3^{\frac{3}{4}}\right)^4$	13. $(16x^{24})^{0.5}$	14. $\left(\frac{16x^3}{81}\right)^{\frac{1}{4}}$	15. $(\sqrt[3]{x^4})^6$

III. Quotient Rule

Rule: $\frac{a^m}{a^n} =$ _____

Examples:

16. $\frac{x^5}{x^2}$	17. $\frac{8x^3}{4x}$	18. $\frac{5^{\frac{7}{3}}}{5^{\frac{1}{3}}}$	19. $\frac{\sqrt[3]{b^5}}{b^{\frac{4}{3}}}$
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IV. Negative Exponent Rule

Rules: $b^{-n} = $	_	$\frac{1}{b^{-n}} = \underline{\qquad}$	
Examples:			
20. 4 ⁻²	21. $\frac{5}{x^{-7}}$	22. $\frac{x^{-1/2}}{x^{2.5}}$	23. $\frac{x^{-1/3}}{\sqrt[4]{x^3}}$

V. Putting it all together

24.
$$(4x^{\frac{3}{4}}y^4)^{\frac{1}{2}}$$
 25. $\left(\frac{16x^{\frac{1}{6}y^{-2}}}{x^{\frac{-1}{6}y^6}}\right)^{\frac{3}{2}}$ 26. $\left(\frac{25y^{-4}}{x^{-\frac{1}{2}y^6}}\right)^{-\frac{1}{2}}$

VI. Use DESMOS to solve each rational equation.

27.
$$8^{\frac{2}{3}} = 4^{y}$$
 28. $16 = 8^{x}$ 29. $125 = 25^{\frac{x}{3}}$

30.
$$\sqrt[4]{64} = 4^{x+2}$$
 31. $9^{2r} = 27^{r-2}$

VII. Determine if each statement is true or fase.

32.
$$\sqrt{32} = 2^{\frac{5}{2}}$$
 33. $16^{\frac{3}{2}} = 8^2$ 34. $4^{\frac{1}{2}} = \sqrt[4]{64}$

35.
$$2^8 = \left(\sqrt[3]{16}\right)^6$$
 36. $\left(\sqrt{64}\right)^{\frac{1}{3}} = 8^{\frac{1}{6}}$

VIII. Rewrite each rational exponent as a radical then solve.

37. $(2x-9)^{\frac{1}{2}} = 25$ 38. $(x+5)^{\frac{1}{2}} = (2x-3)^{\frac{1}{2}}$ 39. $(x-6)^{\frac{1}{2}} = 4$

Hint: $\sqrt{2x - 9} = 25$

Unit 8 Part 2 Lesson 6 Homework

Simplify Using your Exponent Rules

1.
$$(8^2)^{\frac{1}{3}}$$
 2. $(x^{\frac{1}{5}})^0$ 3. $(2c^{\frac{2}{3}})^6$ 4. $(c^{\frac{1}{5}}d^{-\frac{4}{3}})^{-15}$

5.
$$(81x^{12})^{0.75}$$
 6. $(64x^4)^{\frac{3}{2}}$ 7. $\frac{b^{\frac{1}{3}}}{\sqrt[3]{b}}$ 8. $(b^{\frac{1}{2}})^2$

9.
$$\left(\sqrt[3]{x^2}\right)^6$$
 10. $\frac{\left(4\sqrt{x}\right)^2}{(2x)^5}$ 11. $\left(9a^6b^{-4}\right)^{-\frac{1}{2}}$ 12. $\frac{5\sqrt{b^3}}{b^{\frac{4}{3}}}$

13. Multiple Choice: Which expression is equivalent to: $(8w^7x^{-5}y^3z^{-9})^{-\frac{2}{3}}$?

A.
$$\frac{x^{\frac{10}{3}}z^6}{4w^{\frac{14}{3}}y^2}$$
 B. $\frac{4w^{\frac{14}{3}}y^2}{x^{\frac{10}{3}}z^6}$ C. $\frac{2w^{\frac{5}{3}}y^{\frac{1}{3}}}{x^{\frac{7}{3}}z^{\frac{11}{3}}}$ D. $\frac{x^{\frac{7}{3}}z^{\frac{11}{3}}}{2w^{\frac{5}{3}}y^{\frac{1}{3}}}$

14. Multiple Choice: Rewrite as a rational exponent $\left(\sqrt[a]{b^c}\right)^d$

A.
$$b^{\frac{ac}{a}}$$
 B. $b^{\frac{ad}{c}}$ C. $b^{\frac{cd}{a}}$ D. b^{acd}

Rewrite the rational exponent then solve the equation.

15.
$$32 = n^{\frac{1}{2}} + 24$$
 16. $(m+5)^{\frac{1}{2}} = (2m-7)^{\frac{1}{2}}$ 17. $n = (30-n)^{\frac{1}{2}}$

Unit 8 Lesson 7 Homework

Name: _____

Simplify. Leave your answer as rational exponents if necessary.

1.
$$\sqrt[4]{(16x)^5}$$
2. $(\sqrt[3]{27x^9})^{-2}$ 3. $\sqrt[3]{-8x^9}$ 4. $(\frac{\sqrt{bc}}{3ab^{-1}c^2})^{-2}$ 5. $\frac{\sqrt[3]{27x^3}}{(16x)^{\frac{1}{4}}}$ 6. $(-1000p^3)^{\frac{2}{3}}$ 7. $\sqrt[4]{(81x^4)^5}$ 8. $2(36a^{-3}b^8)^{0.5}$ 9. $4\sqrt{x^3} \cdot \sqrt[3]{x^2}$ 10. $\frac{\sqrt[3]{-16b^5}}{(2b)^{\frac{4}{3}}}$ 11. $(27p^6)^{-\frac{5}{3}}$ 12. $(9r^4)^{0.5}$

$$13. \left(\frac{16x^{-4}}{81y^{18}}\right)^{0.5} 14. \left(x^{-2}w^{\frac{1}{6}}\right) \cdot \left(25x^{\frac{1}{2}}w\right)^{-1} 15. \left(\frac{64a^2b^{-\frac{1}{2}}c^0}{125abc}\right)^{-\frac{1}{3}}$$

Rewrite using rational exponents. Do not simplify.

16. $(\sqrt{2x})^5$ 17. $(\sqrt[3]{-7x^2y})^2$ 18. $\sqrt[4]{9x^2}$ 19. $\sqrt[b]{(wxy)^c}$

Rewrite using radicals. Do not simplify.

20. $(3x)^{\frac{4}{3}}$ 21. $(3x)^{2.5}$ 22. $(-27x^3y)^{\frac{2}{5}}$ 23. $p(rs^3)^{-\frac{1}{2}}$

Solve using Desmos.

24.
$$\sqrt[4]{81} = 27^x$$
 25. $8^{\frac{x}{3}} = (\sqrt[3]{64})^2$ 26. $25^{2m} = 125^{m-3}$

Unit 8 Lesson 8: Rational Functions			
I. Graph of $y = \frac{1}{r}$			
Our last function to study in math 2 is the	ne functio	on. Rational means "	".
Complete the table to help you graph	the parent function $y = \frac{1}{x}$.	X Y	
		3	
-2			
-1		1	
0			
0.5		<	
1			$2 \qquad 3 \qquad x$
2			
The shape of the graph is called a	•	-2-2	
Each curve is called a			
Notice that the branches of this parent	graph are in Quadrants	and	
Unlike the other functions we have stu	idiad rational functions are not		functions
Unite the other functions we have su	iuleu, fational functions are not		
They have	, which are lines that the	curve approaches but never actua	lly touches. These are
places where the function is undefine	d.		
The parent function has two asymptot	A 6 ·		
The parent function has two asymptot	55.		
Horizontal asymptot	:e:		
• Vertical asymptote:			
Other Key Features:			
Domain:	Increasing:	x-intercept:	
Range:	Decreasing:	y-intercept:	
II. Transformations of $y = \frac{1}{x}$			
The graph form of a rational function is with square root functions and quadratic	$y = \frac{a}{x-h} + k$. The effects of a_{x-h} functions.	, <i>h</i> , and <i>k</i> are the same verbal desc	criptions that we learned
*Special Note: A vertical compression is	written as $y = \frac{1}{a(x-k)} + k \operatorname{since}$	e the a value is a fraction between	0 and 1.
Example: Determine the vertical compr	u(x-n)	$f(r) = \frac{-1}{r}$	$y = \frac{1}{1}$
Example. Determine the vertical comple	$3x^{-3x}$	$f(x) = \frac{4x}{4x}$	$y = \frac{1}{5(x-2)}$
1. Write the equation of the rational fun- key features.	ction that is reflected over the x-	-axis, shifted right 2 units and up 5	5 units. Then identify the
Equation:			
Domain:	Increasing:	x-intercept:	
Danger	Dearcasin		
Kange:	Decreasing:	y-intercept:	

2. Describe	the transfor	mations and	identify the key features that occurred from the	parent function $f(x) = \frac{-1}{2(x+4)} - 1$
Transforma	tions:			
Domain:			Increasing:	x-intercept:
Range:			Decreasing:	y-intercept:
III. Inverse	Variation -	- A Very Spo	ecial Rational Function!	
An left or right)	. An inverse	variation ca	equation is a specific rational functi n only have a vertical or vertic	ion that is not translated (not shifted up, down, cal
Let's look at	the relation	ship betweer	the table and the equation of a rational function	n that is also an inverse variation equation.
The parent :	function is y	$=\frac{1}{x}$. If we	cross multiply, we get	
Add a colum x	nn titled <i>xy :</i> <i>y</i>	and fill it in.	What did you observe?	
-2			The product of <i>xy</i> is known as, or the	e constant of variation.
-0.5			This constant of variation is the same as the ve	ertical stretch or compression.
0.5			We can write an equation for the inverse variat	tion by finding the constant of variation. $y = \frac{k}{x}$

1. Fill in the table and then write an equation.

x	у
-2	-2
-1	-4
0	Undefined
1	4
2	2

1

2. Use the table to complete the questions.

x	у
-2	-4
-1	-8
0	Undefined
1	8
2	4

A. Determine the constant of variation, k.

B. Write an equation to represent the function.



3. Different representations are shown below for f(x), h(x), and m(x).



4. The function $f(x) = \frac{k}{x}$ is transformed to create g(x). Which of the following transformations would result in an inverse variation equation? Circle all that apply.

- A. A vertical stretch by 2 and horizontal shift up 3.
- C. Translation left 7, down 7



- B. A vertical compression by 1/4
- D. A vertical stretch by 10 and reflect over the x-axis



	x	f(x)
-	2	-1
-	1	-4
0)	Undefined
1		8
2		5

- 5. True or False: The vertical asymptote of an inverse variation is always x = 0.
- 6. Fill in the blank: The horizontal asymptote of every inverse variation equation is _____