Unit 8 Part 2 Lesson 6: Simplifying and Solving with Rational Exponents

## I. Product Rule

Rule: $\left(a^{m}\right)\left(a^{n}\right)=$

Examples:

1. $\left(x^{3}\right)\left(x^{5}\right)$
2. $\left(m^{4}\right)(m)$
3. $\left(2 y^{2}\right)\left(-4 y^{5}\right)$
4. $\left(x^{\frac{1}{2}}\right)\left(x^{\frac{3}{2}}\right)$
5. $\left(7^{\frac{3}{4}}\right)\left(7^{\frac{1}{4}}\right)$
6. $\left(5 m^{\frac{1}{3}}\right)\left(-m^{\frac{4}{3}}\right)$
7. $(\sqrt{x})\left(\sqrt[2]{x^{3}}\right)$

## II. Power Rules

Rules: $\left(a^{m}\right)^{n}=$ $\qquad$

$$
(a b)^{m}=
$$

$\left(\frac{a}{b}\right)^{m}=$ $\qquad$

Examples:
8. $\left(x^{2}\right)^{3}$
9. $\left(2 x^{2} y\right)^{3}$
10. $\left(\frac{x}{y}\right)^{5}$
11. $\left(\frac{x}{3 y^{2}}\right)^{4}$
12. $\left(3^{\frac{3}{4}}\right)^{4}$
13. $\left(16 x^{24}\right)^{0.5}$
14. $\left(\frac{16 x^{3}}{81}\right)^{\frac{1}{4}}$
15. $\left(\sqrt[3]{x^{4}}\right)^{6}$

## III. Quotient Rule

Rule: $\frac{a^{m}}{a^{n}}=$
Examples:
16. $\frac{x^{5}}{x^{2}}$
17. $\frac{8 x^{3}}{4 x}$
18. $\frac{5^{\frac{7}{3}}}{5^{\frac{1}{3}}}$
19. $\frac{\sqrt[3]{b^{5}}}{b^{\frac{4}{3}}}$
IV. Negative Exponent Rule

Rules: $b^{-n}=$ $\qquad$

$$
\frac{1}{b^{-n}}=
$$

Examples:
20. $4^{-2}$
21. $\frac{5}{x^{-7}}$
22. $\frac{x^{-1 / 2}}{x^{2.5}}$
23. $\frac{x^{-1 / 3}}{\sqrt[4]{x^{3}}}$
V. Putting it all together
24. $\left(4 x^{\frac{3}{4}} y^{4}\right)^{\frac{1}{2}}$
25. $\left(\frac{16 x^{\frac{1}{6}} y^{-2}}{x^{\frac{-1}{6}} y^{6}}\right)^{\frac{3}{2}}$
26. $\left(\frac{25 y^{-4}}{x^{-\frac{1}{2}} y^{6}}\right)^{-\frac{1}{2}}$
VI. Use DESMOS to solve each rational equation.
27. $8^{\frac{2}{3}}=4^{y}$
28. $16=8^{x}$
29. $125=25^{\frac{x}{3}}$
30. $\sqrt[4]{64}=4^{x+2}$
31. $9^{2 r}=27^{r-2}$
VII. Determine if each statement is true or fase.
32. $\sqrt{32}=2^{\frac{5}{2}}$
33. $16^{\frac{3}{2}}=8^{2}$
34. $4^{\frac{1}{2}}=\sqrt[4]{64}$
35. $2^{8}=(\sqrt[3]{16})^{6}$
36. $(\sqrt{64})^{\frac{1}{3}}=8^{\frac{1}{6}}$
VIII. Rewrite each rational exponent as a radical then solve.
37. $(2 x-9)^{\frac{1}{2}}=25$
38. $(x+5)^{\frac{1}{2}}=(2 x-3)^{\frac{1}{2}}$
39. $(x-6)^{\frac{1}{2}}=4$

Hint: $\sqrt{2 x-9}=25$
$\qquad$

## Simplify Using your Exponent Rules

1. $\left(8^{2}\right)^{\frac{1}{3}}$
2. $\left(x^{\frac{1}{5}}\right)^{0}$
3. $\left(2 c^{\frac{2}{3}}\right)^{6}$
4. $\left(c^{\frac{1}{5}} d^{-\frac{4}{3}}\right)^{-15}$
5. $\left(81 x^{12}\right)^{0.75}$
6. $\left(64 x^{4}\right)^{\frac{3}{2}}$
7. $\frac{b^{\frac{1}{3}}}{\sqrt[3]{b}}$
8. $\left(b^{\frac{1}{2}}\right)^{2}$
9. $\left(\sqrt[3]{x^{2}}\right)^{6}$
10. $\frac{(4 \sqrt{x})^{2}}{(2 x)^{5}}$
11. $\left(9 a^{6} b^{-4}\right)^{-\frac{1}{2}}$
12. $\frac{5 \sqrt{b^{3}}}{b^{\frac{4}{3}}}$
13. Multiple Choice: Which expression is equivalent to: $\left(8 w^{7} x^{-5} y^{3} z^{-9}\right)^{-\frac{2}{3}}$ ?
A. $\frac{x^{\frac{10}{3}} z^{6}}{4 w^{\frac{14}{3}} y^{2}}$
B. $\frac{4 w^{\frac{14}{3}} y^{2}}{x^{\frac{10}{3}} z^{6}}$
C. $\frac{2 w^{\frac{5}{3}} y^{\frac{1}{3}}}{x^{\frac{7}{3}} z^{\frac{11}{3}}}$
D. $\frac{x^{\frac{7}{3}} z^{\frac{11}{3}}}{2 w^{\frac{5}{3}} y^{\frac{1}{3}}}$
14. Multiple Choice: Rewrite as a rational exponent $\left(\sqrt[a]{b^{c}}\right)^{d}$
A. $b^{\frac{a c}{d}}$
B. $b^{\frac{a d}{c}}$
C. $b^{\frac{c d}{a}}$
D. $b^{a c d}$

Rewrite the rational exponent then solve the equation.
15. $32=n^{\frac{1}{2}}+24$
16. $(m+5)^{\frac{1}{2}}=(2 m-7)^{\frac{1}{2}}$
17. $n=(30-n)^{\frac{1}{2}}$
$\qquad$
Simplify. Leave your answer as rational exponents if necessary.

1. $\sqrt[4]{(16 x)^{5}}$
2. $\left(\sqrt[3]{27 x^{9}}\right)^{-2}$
3. $\sqrt[3]{-8 x^{9}}$
4. $\left(\frac{\sqrt{b c}}{3 a b^{-1} c^{2}}\right)^{-2}$
5. $\frac{\sqrt[3]{27 x^{3}}}{(16 \mathrm{x})^{\frac{1}{4}}}$
6. $\left(-1000 p^{3}\right)^{\frac{2}{3}}$
7. $\sqrt[4]{\left(81 \mathrm{x}^{4}\right)^{5}}$
8. $2\left(36 a^{-3} b^{8}\right)^{0.5}$
9. $4 \sqrt{\mathrm{x}^{3}} \cdot \sqrt[3]{\mathrm{x}^{2}}$
10. $\frac{\sqrt[3]{-16 b^{5}}}{(2 b)^{\frac{4}{3}}}$
11. $\left(27 p^{6}\right)^{-\frac{5}{3}}$
12. $\left(9 r^{4}\right)^{0.5}$
13. $\left(\frac{16 x^{-4}}{81 y^{18}}\right)^{0.5}$
14. $\left(x^{-2} w^{\frac{1}{6}}\right) \cdot\left(25 x^{\frac{1}{2}} w\right)^{-1}$
15. $\left(\frac{64 a^{2} b^{-\frac{1}{2}} c^{0}}{125 a b c}\right)^{-\frac{1}{3}}$

Rewrite using rational exponents. Do not simplify.
16. $(\sqrt{2 \mathrm{x}})^{5}$
17. $\left(\sqrt[3]{-7 x^{2} y}\right)^{2}$
18. $\sqrt[4]{9 \mathrm{x}^{2}}$
19. $\sqrt[b]{(w x y)^{c}}$

Rewrite using radicals. Do not simplify.
20. $(3 x)^{\frac{4}{3}}$
21. $(3 x)^{2.5}$
22. $\left(-27 x^{3} y\right)^{\frac{2}{5}}$
23. $\mathrm{p}\left(\mathrm{rs}^{3}\right)^{-\frac{1}{2}}$

Solve using Desmos.
24. $\sqrt[4]{81}=27^{x}$
25. $8^{\frac{x}{3}}=(\sqrt[3]{64})^{2}$
26. $25^{2 \mathrm{~m}}=125^{\mathrm{m}-3}$

## Unit 8 Lesson 8: Rational Functions

I. Graph of $\boldsymbol{y}=\frac{1}{x}$

Our last function to study in math 2 is the $\qquad$ function. Rational means " $\qquad$ ".
Complete the table to help you graph the parent function $y=\frac{1}{x}$.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| -0.5 |  |
| 0 |  |
| 0.5 |  |
| 1 |  |
| 2 |  |



The shape of the graph is called a $\qquad$ .

Each curve is called a $\qquad$ _.

Notice that the branches of this parent graph are in Quadrants $\qquad$ and $\qquad$ .

Unlike the other functions we have studied, rational functions are not $\qquad$ functions.

They have $\qquad$ , which are lines that the curve approaches but never actually touches. These are places where the function is undefined.

The parent function has two asymptotes:

- Horizontal asymptote: $\qquad$
- Vertical asymptote: $\qquad$
Other Key Features:
Domain: $\qquad$ Increasing: $\qquad$ x-intercept: $\qquad$
Range: $\qquad$ Decreasing: $\qquad$ $y$-intercept: $\qquad$


## II. Transformations of $y=\frac{1}{x}$

The graph form of a rational function is $y=\frac{a}{x-h}+k$. The effects of $a, h$, and $k$ are the same verbal descriptions that we learned with square root functions and quadratic functions.
*Special Note: A vertical compression is written as $y=\frac{1}{a(x-h)}+k$ since the $a$ value is a fraction between 0 and 1.
Example: Determine the vertical compression. $y=\frac{2}{3 x} \quad f(x)=\frac{-1}{4 x} \quad y=\frac{1}{5(x-2)}$

1. Write the equation of the rational function that is reflected over the $x$-axis, shifted right 2 units and up 5 units. Then identify the key features.

Equation:

Domain: $\qquad$ Increasing: $\qquad$ x-intercept: $\qquad$

Range: $\qquad$ Decreasing: $\qquad$ y-intercept: $\qquad$
2. Describe the transformations and identify the key features that occurred from the parent function $f(x)=\frac{-1}{2(x+4)}-1$

Transformations:

Domain: $\qquad$ Increasing: $\qquad$ x-intercept: $\qquad$

Range: $\qquad$ Decreasing: $\qquad$ y-intercept: $\qquad$

## III. Inverse Variation - A Very Special Rational Function!

An $\qquad$ equation is a specific rational function that is not translated (not shifted up, down, left or right). An inverse variation can only have a vertical $\qquad$ or vertical $\qquad$ —.

Let's look at the relationship between the table and the equation of a rational function that is also an inverse variation equation.
The parent function is $y=\frac{1}{x}$. If we cross multiply, we get $\qquad$ .

Add a column titled $x y$ and fill it in.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |  |
| :---: | :---: | :---: |
| -2 |  |  |
| -1 |  |  |
| -0.5 |  |  |
| 0 |  |  |
| 0.5 |  |  |
| 1 |  |  |
| 2 |  |  |

What did you observe?

The product of $x y$ is known as $\qquad$ or the constant of variation.

This constant of variation is the same as the vertical stretch or compression.

We can write an equation for the inverse variation by finding the constant of variation. $\quad y=\frac{k}{x}$

1. Fill in the table and then write an equation.

| $x$ | $y$ |
| :--- | :--- |
| -2 | -2 |
| -1 | -4 |
| 0 | Undefined |
| 1 | 4 |
| 2 | 2 |

2. Use the table to complete the questions.

| $x$ | $y$ |
| :--- | :--- |
| -2 | -4 |
| -1 | -8 |
| 0 | Undefined |
| 1 | 8 |
| 2 | 4 |

A. Determine the constant of variation, $k$.
B. Write an equation to represent the function.


3. Different representations are shown below for $f(x), h(x)$, and $m(x)$.

| $x$ | $f(x)$ |
| :--- | :--- |
| -2 | -0.25 |
| -1 | -0.5 |
| 0 | Undefined |
| 1 | 0.5 |
| 2 | 0.25 |



$$
m(x)=\frac{3}{x}
$$

A. Write the equation for $f(x)$.
B. Write the equation for $h(x)$.
C. Which function has the greatest constant of proportionality? $\qquad$
4. The function $f(x)=\frac{k}{x}$ is transformed to create $g(x)$. Which of the following transformations would result in an inverse variation equation? Circle all that apply.
A. A vertical stretch by 2 and horizontal shift up 3 .
B. A vertical compression by $1 / 4$
C. Translation left 7, down 7
D. A vertical stretch by 10 and reflect over the x -axis

F.

G.

H.

| $x$ | $f(x)$ |
| :---: | :--- |
| -2 | -1 |
| -1 | -4 |
| 0 | Undefined |
| 1 | 8 |
| 2 | 5 |

5. True or False: The vertical asymptote of an inverse variation is always $x=0$.
6. Fill in the blank: The horizontal asymptote of every inverse variation equation is $\qquad$ .
