Unit 8 Part 2 Lesson 6: Simplifying and Solving with Rational Exponents

I. Product Rule

Rule: $(a^m)(a^n) =$ a^{m+n}

Examples:

1. $(x^3)(x^5)$
   x^8
2. $(m^4)(m)$
   m^5
3. $(2y^2)(-4y^5)$
   $-8y^7$
4. $(x^{\frac{1}{2}})(x^{\frac{3}{2}})$
   $x^{4/2} \Rightarrow x^2$
5. $(7^{\frac{3}{4}})(7^{\frac{1}{4}})$
   $7^{4/4} \Rightarrow 7$
6. $(5m^{\frac{1}{3}})(-m^{\frac{4}{3}})$
   $-5m^{5/3}$
7. $(\sqrt{x})(\sqrt[2]{x^3})$
   $x^{1/2}\ x^{3/2}$
   $x^{4/2} \Rightarrow x^2$

II. Power Rules

Rules: $(a^m)^n =$ a^{mn} $\quad (ab)^m =$ $a^m b^m$ $\quad \left(\frac{a}{b}\right)^m =$ $\frac{a^m}{b^m}$

Examples:

8. $(x^2)^3$
   x^6
9. $(2x^2y)^3$
   $2^3x^6y^3$
   $8x^6y^3$
10. $\left(\frac{x}{y}\right)^5$
    x^5/y^5
11. $\left(\frac{x}{3y^2}\right)^4$
    $\frac{x^4}{3^4y^8} \Rightarrow \frac{x^4}{81y^8}$
12. $\left(3^{\frac{3}{4}}\right)^4$
    $3^{12/4} \Rightarrow 3^3 \Rightarrow 27$
13. $(16x^{24})^{\cancel{0.5}\ 1/2}$
    $16^{1/2}x^{12}$
    $4x^{12}$
14. $\left(\frac{16x^3}{81}\right)^{\frac{1}{4}}$
    $\frac{16^{1/4}\ x^{3/4}}{81^{1/4}}$
    $\frac{2x^{3/4}}{3}$
15. $(\sqrt[3]{x^4})^6$
    $(x^{4/3})^6 \Rightarrow x^{24/3} = x^8$

III. Quotient Rule

Rule: $\frac{a^m}{a^n} =$ a^{m-n}

Examples:

16. $\frac{x^5}{x^2}$
    $x^{5-2} \Rightarrow x^3$
17. $\frac{8x^3}{4x}$
    $2x^{3-1} \Rightarrow 2x^2$
18. $\frac{5^{\frac{7}{3}}}{5^{\frac{1}{3}}}$
    $5^{7/3-1/3} = 5^{6/3} = 5^2$
19. $\frac{\cancel{\sqrt[3]{b^5}}}{b^{\frac{4}{3}}}$ $b^{5/3}$
    $b^{5/3-4/3} = b^{1/3}$

IV. Negative Exponent Rule

Rules: $b^{-n} =$ $1/b^n$ $\quad \frac{1}{b^{-n}} =$ b^n

Examples:

20. 4^{-2}
    $\frac{1}{4^2} \Rightarrow \frac{1}{16}$
21. $\frac{5}{x^{-7}}$
    $5x^7$
22. $\frac{x^{-1/2}}{\cancel{x^{2.5}}}$ $x^{5/2}$
    $\frac{1}{x^{1/2}x^{5/2}} \Rightarrow \frac{1}{x^{6/2}}$
    $\frac{1}{x^3}$
23. $\frac{x^{-1/3}}{\cancel{\sqrt[4]{x^3}}}$ $x^{3/4}$
    $x^{-1/3-3/4} \Rightarrow x^{-13/12}$
    $\frac{1}{x^{13/12}}$

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V. Putting it all together

24. $(4x^{\frac{3}{4}}y^{4})^{\frac{1}{2}}$

$4^{1/2}(x^{3/4})^{1/2}(y^{4})^{1/2}$

$4^{1/2}x^{3/8}y^{4/2}$

$2x^{3/8}y^{2}$

25. $\left(\frac{16x^{\frac{1}{6}}y^{-2}}{x^{\frac{-1}{6}}y^{6}}\right)^{\frac{3}{2}}$

$\frac{16^{3/2}(x^{1/6})^{3/2}(y^{-2})^{3/2}}{(x^{-1/6})^{3/2}(y^{6})^{3/2}}$

$\frac{16^{3/2}x^{3/12}y^{-6/2}}{x^{-3/12}y^{18/2}} \Rightarrow \frac{16^{3/2}x^{1/4}y^{-3}}{x^{-1/4}y^{9}}$

$\frac{16^{3/2}x^{1/4}x^{1/4}}{y^{3}y^{9}} = \frac{64x^{2/4}}{y^{12}} = \frac{64x^{1/2}}{y^{12}}$

26. $\left(\frac{25y^{-4}}{x^{-\frac{1}{2}}y^{6}}\right)^{-\frac{1}{2}}$

VI. Use DESMOS to solve each rational equation.

27. $8^{\frac{2}{3}} = 4^{x}$

$x = 1$

28. $16 = 8^{x}$

$x = 4/3$

29. $125 = 25^{\frac{x}{3}}$

$x = 4.5$

30. $\sqrt[4]{64} = 4^{x+2}$

$x = -1.25$

31. $9^{2r} = 27^{r-2}$

$(3^{2})^{2r} = (3^{3})^{r-2}$

$4r = 3r - 6$

$r = -6$

VII. Determine if each statement is true or fase.

32. $\sqrt{32} = 2^{\frac{5}{2}}$

True

33. $16^{\frac{3}{2}} = 8^{2}$

True

34. $4^{\frac{1}{2}} = \sqrt[4]{64}$

False

35. $2^{8} = (\sqrt[3]{16})^{6}$

True

36. $(\sqrt{64})^{\frac{1}{3}} = 8^{\frac{1}{6}}$

False

VIII. Rewrite each rational exponent as a radical then solve.

37. $(2x-9)^{\frac{1}{2}} = 25$

Hint: $\sqrt{2x-9} = 25$

$2x - 9 = 625$

$\frac{2x}{2} = \frac{634}{2}$

$x = 317$

$\{317\}$

38. $(x+5)^{\frac{1}{2}} = (2x-3)^{\frac{1}{2}}$

$\sqrt{x+5} = \sqrt{2x-3}$

$x + 5 = 2x - 3$

$8 = x$

$\{8\}$

39. $(x-6)^{\frac{1}{2}} = 4$

$\sqrt{x-6} = 4$

$x - 6 = 16$

$x = 22$

$\{22\}$

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