Unit 8 Lesson 1 – Solving Radical Equations

Name: Key

Radical Equation: An equation with a Variable inside the Radical.

Example: $\sqrt{x+3} = 5$

When we solve equations in math we use Inverse operations to help us get the Variable by itself.

Steps:

- Isolate the square root
- Undo the square root.
  - How do you UNDO a square root? Square
- Solve for x. Depending on the problem, this might be solving a Linear equation or a Quadratic equation. What method(s) might be the best choices to solve a quadratic equation? → • Factor • QF • Complete □ • Sq. Root.
- Ensure you do not have any extraneous solutions.
  - How do we make sure all of our solutions are correct? CHECK!

Examples:

1. $(\sqrt{x})^2 = (8)^2$ $\{64\}$
   $x = 64$
   ✓: $\sqrt{64} = 8$
   $8 = 8$ ✓

2. $(\sqrt{x+3})^2 = (5)^2$ $\{22\}$
   $x + 3 = 25$
   $x = 22$
   ✓: $\sqrt{22+3} = 5$
   $\sqrt{25} = 5$
   $5 = 5$ ✓

3. $\sqrt{x} + 9 = 3$ $\{\ \}$ or ϕ
   $-9 \quad -9$
   $(\sqrt{x})^2 = (-6)^2$
   $x = 36$
   ✓: $\sqrt{36} + 9 = 3$
   $6 + 9 = 3$
   $15 \neq 3$

4. $\sqrt{3x} + 5 = 17$ $\{48\}$
   $(\sqrt{3x})^2 = (12)^2$
   $3x = 144$
   $x = 48$
   ✓: $\sqrt{48 \cdot 3} + 5 = 17$
   $\sqrt{144} + 5 = 17$
   $12 + 5 = 17$
   $17 = 17$ ✓

5. $\frac{2\sqrt{x-1}}{2} = \frac{18}{2}$ $\{82\}$
   $(\sqrt{x-1})^2 = (9)^2$
   $x - 1 = 81$
   $x = 82$
   ✓: $2\sqrt{82-1} = 18$
   $2\sqrt{81} = 18$
   $2(9) = 18$ ✓

6. $\left(\sqrt{\frac{x}{5}}\right)^2 = (3)^2$ $\{45\}$
   $x/5 = 9$
   $x = 45$
   ✓: $\sqrt{\frac{45}{5}} = 3$
   $\sqrt{9} = 3$
   $3 = 3$ ✓

7. $(\sqrt{x+2})^2 = (\sqrt{x-6})^2$ $\{\ \}$ or ϕ
   $x + 2 = x - 6$
   $2 \neq 6$

8. $(\sqrt{x})^2 = (\sqrt{2x-7})^2$ $\{7\}$
   $x = 2x - 7$
   $-x = -7$
   $x = 7$
   ✓: $\sqrt{7} = \sqrt{2(7)-7}$
   $\sqrt{7} = \sqrt{14-7}$
   $\sqrt{7} = \sqrt{7}$ ✓

9. $(\sqrt{x^2-5})^2 = (\sqrt{x+1})^2$ $\{3, \cancel{-2}\}$
   $x^2 - 5 = x + 1$
   $x^2 - x - 6 = 0$
   $(x-3)(x+2) = 0$
   $x = 3, -2$
   ✓: $\sqrt{9-5} = \sqrt{3+1}$
   $\sqrt{4} = \sqrt{4}$ ✓
   ✓: $\sqrt{4-5} = \sqrt{-2+1}$
   $\sqrt{-1} = \sqrt{-1}$ ✓

Examples:

1. $(x)^2 = (\sqrt{56-x})^2$ $\{7\}$

$x^2 = 56 - x$

$x^2 + x - 56 = 0$

$(x+8)(x-7) = 0$

$x = -8, 7$

✓: $-8 = \sqrt{56+8}$ $\quad$ $7 = \sqrt{56-7}$

$-8 \neq \sqrt{64}$ $\quad$ $7 = \sqrt{49}$

$-8 \neq 8$ $\quad$ $7 = 7$ ✓

3. $\sqrt{-14+9x} - x = 0$

$+x \quad +x$

$(\sqrt{-14+9x})^2 = (x)^2$ $\{7, 2\}$

$-14 + 9x = x^2$

$-14 = x^2 - 9x$

$x^2 - 9x + 14 = 0$

$(x-7)(x-2) = 0$

$x = 7$ or 2

5. $(x-6)^2 = (\sqrt{21-4x})^2$ $\{\ \}$

$(x-6)(x-6) = 21 - 4x$

$x^2 - 12x + 36 = 21 - 4x$

$x^2 - 8x - 15 = 0$

$(x-5)(x-3) = 0$

$x = 5, 3$

* Yuck and all that work with no solution!

You Try:

2. $(x)^2 = (\sqrt{2x})^2$ $\{0, 2\}$

$x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

✓: $0 = \sqrt{2(0)}$ $\quad$ $2 = \sqrt{2 \cdot 2}$

$0 = \sqrt{0}$ $\quad$ $2 = \sqrt{4}$

$0 = 0$ $\quad$ $2 = 2$

4. $\sqrt{-7+8x} - x = 0$ $\{7, 1\}$

$(\sqrt{-7+8x})^2 = (x)^2$

$-7 + 8x = x^2$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$x = 7, 1$

Is there a way to find the solution to these equations using DESMOS?

5. $x - 6 = \sqrt{21 - 4x}$

Take the left side of the equation and set it equal to y. What type of function is this? Linear

Take the right side of the equation and set it equal to y. What type of function is this? Square root

Do these functions intersect?

If not, there is NO SOLUTION $(\emptyset)$

If they intersect, the x value of the point of intersection is the solution. (x, y)

6. $\sqrt{6x - 29} = x - 4$ $\{9, 5\}$

7. $x = \sqrt{4x - 24} + 6$ $\{10, 6\}$

8. $-x + \sqrt{6x - 17} = -2$ $\{7, 3\}$

9. $5\sqrt{x - 4} - x = 0$ $\{5, 20\}$

10. $2\sqrt{3x - 4} - 7 = x - 7$ $\{1.528, 10.472\}$