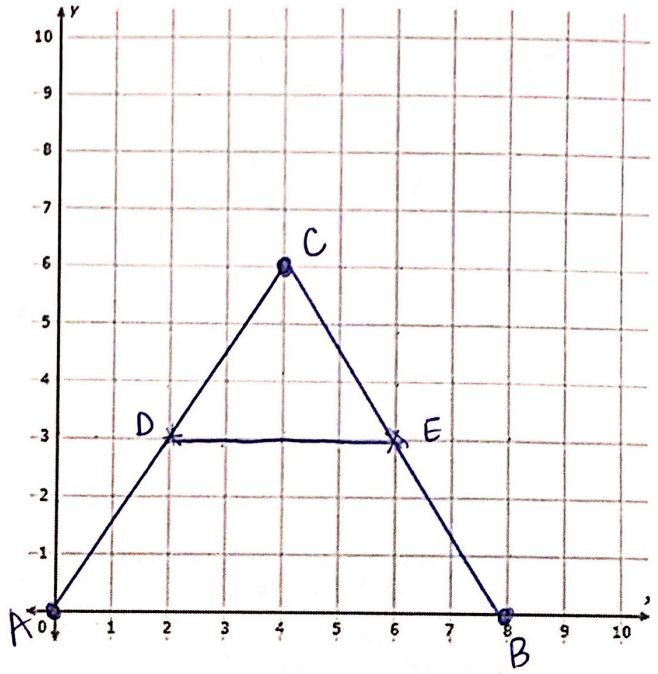
Unit 6 Lesson 6 Notes: Triangle Mid-Segment Theorem and Proportionality Theorem

Triangle Mid-Segment Investigation:

1. Graph ΔABC where $A(0,0)$, $B(8,0)$, and $C(4,6)$. Label each point.



2. Find the midpoint of $\overline{AC}$. Label this point D.

3. Find the midpoint of $\overline{CB}$. Label this point E.

4. Using a ruler connect point D and point E.

5. Determine the length of $\overline{DE}$ and $\overline{AB}$ and record each below.

$\overline{DE}$ = 4 $\overline{AB}$ = 8

6. What relationship do you notice between the length of $\overline{DE}$ and $\overline{AB}$?

2(DE) = AB

7. Find the slope of $\overline{DE}$ and the slope of $\overline{AB}$ and record each below.

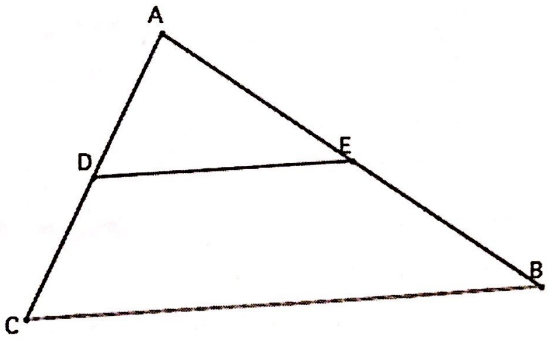
Slope $\overline{DE}$ = 0 Slope $\overline{AB}$ = 0

8. What do you notice about the slope of $\overline{DE}$ and $\overline{AB}$? What does this mean about $\overline{DE}$ and $\overline{AB}$?

They are parallel because their slopes are the same.

Triangle Mid-Segment Theorem: In a triangle, the segment joining the Midpoint of any two sides will be parallel to the third side and HALF its length.

(DE) = ½(AB)
2(DE) = AB



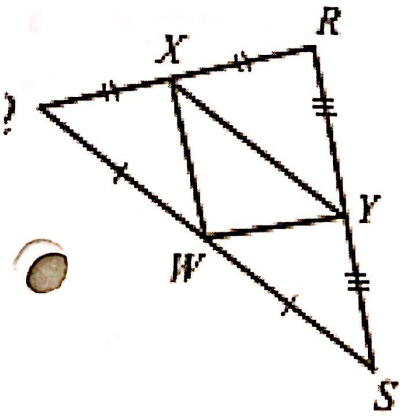
D is the midpoint of $\overline{AC}$ E is the midpoint of $\overline{AB}$

$\overline{DE}$ is parallel to $\overline{CB}$ $DE = \frac{1}{2}CB \Rightarrow 2(DE) = CB$

$\angle ADE \cong \angle ACB$ Because if two parallel lines are cut by a transversal... Corresponding angles are ≅.

$\angle AED \cong \angle ABC$ Because if two parallel lines are cut by a transversal... Corresponding angles are ≅

$\angle DAE \cong \angle CAB$ Because of the Reflexive Property



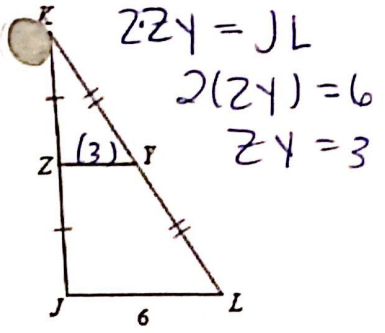
X is the midpoint of $\overline{QR}$ and Y is the midpoint of $\overline{RS}$ so $\overline{XY}$ is parallel to $\overline{QS}$ and QS = 2 XY

W is the midpoint of $\overline{QS}$ and X is the midpoint of $\overline{QR}$ so $\overline{RS}$ is parallel to $\overline{XW}$ and RS = 2 XW

Y is the midpoint of $\overline{RS}$ and W is the midpoint of $\overline{QS}$ so $\overline{WY}$ is parallel to $\overline{QR}$ and QR = 2 WY

Examples:

1. Find ZY

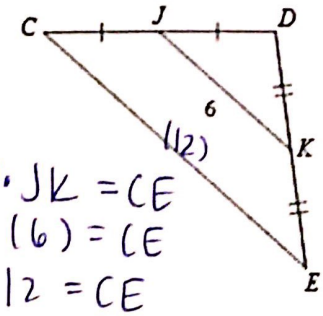


$2ZY = JL$

$2(ZY) = 6$

$ZY = 3$

2. Find CE

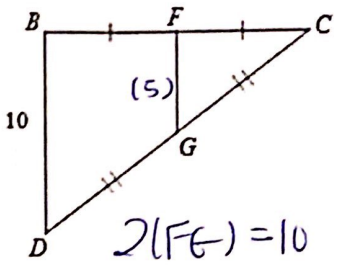


$2 \cdot JK = CE$

$2(6) = CE$

$12 = CE$

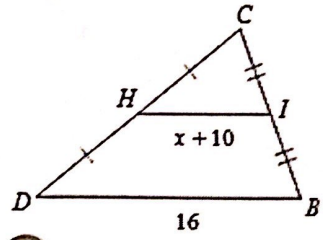
3. Find FG



$2(FG) = 10$

$FG = 5$

4. Solve for x



$2(HI) = DB$

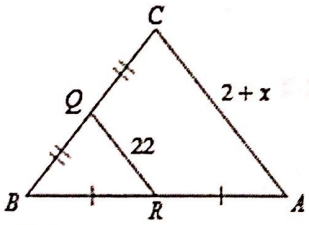
$2(x+10) = 16$

$2x+20 = 16$

$2x = -4$

$x = -2$

5. Solve for x



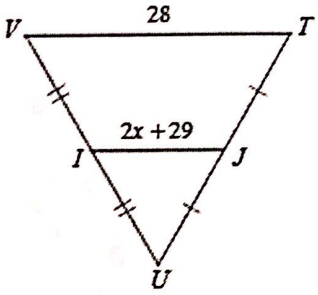
$2(QR) = CA$

$2(22) = 2+x$

$44 = 2+x$

$42 = x$

6. Solve for x



$2(IJ) = VT$

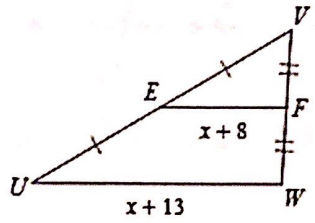
$2(2x+29) = 28$

$4x+58 = 28$

$4x = -30$

$x = -7.5$

7. Find EF



$2(EF) = (UW)$

$2(x+8) = x+13$

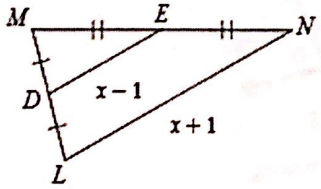
$2x+16 = x+13$

$x = -3$

$EF = (-3)+8$

$= 5$

8. Find LN



$2(DE) = LN$

$2(x-1) = x+1$

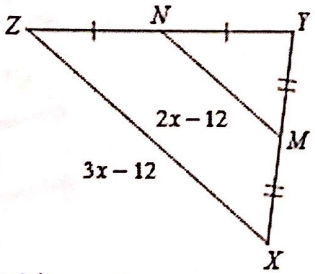
$2x-2 = x+1$

$x = 3$

$LN = (3)+1$

$= 4$

9. Find NM



$2(NM) = ZX$

$2(2x-12) = 3x-12$

$4x-24 = 3x-12$

$x = 12$

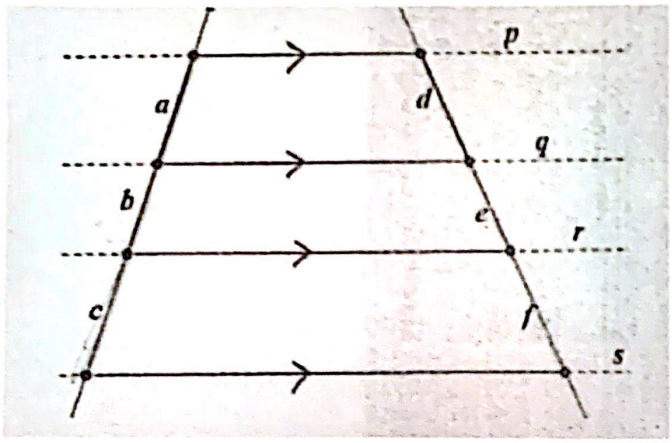
$NN = 2(12)-12$

$= 24-12$

$= 12$

Parallel Proportionality Theorem: If two or more parallel lines are cut by two or more transversals, then the lengths of corresponding segments are proportional

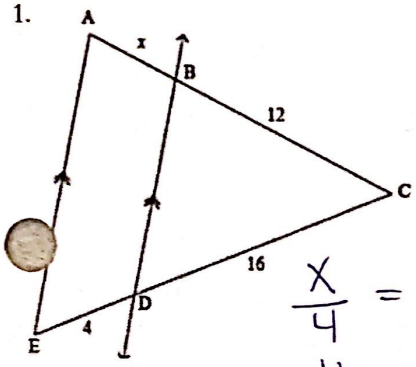
$p \parallel q \parallel r \parallel s$



$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

Examples:

1.

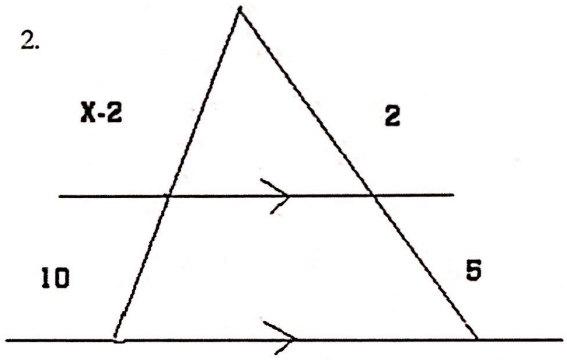


$\frac{x}{4} = \frac{12}{16}$

$16x = 48$

$x = 3$

2.



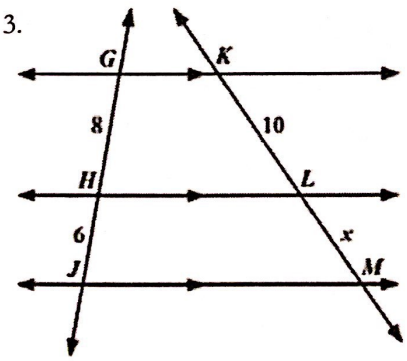
$\frac{2}{x-2} = \frac{5}{10}$

$20 = 5x - 10$

$30 = 5x$

$6 = x$

3.

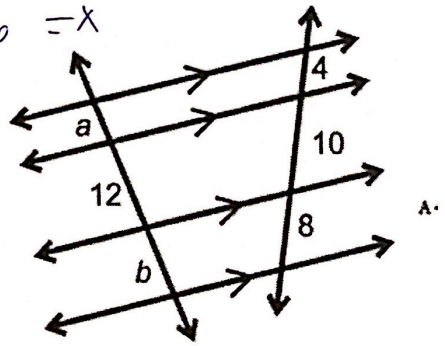


$\frac{8}{10} = \frac{6}{x}$

$8x = 60$

$x = 7.5$

4.



$\frac{a}{4} = \frac{12}{10}$

$10a = 48$

$a = 4.8$

$\frac{12}{10} = \frac{b}{8}$

$96 = 10b$

$b = 9.6$