Unit 6 Lesson 3 Notes Dilations

I. Review

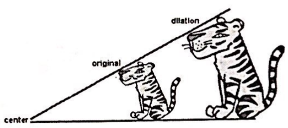
During our last Geometry unit we studied three types of transformations

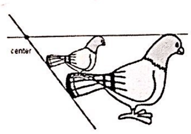
1. Translations
2. Rotations
3. Reflections

All of these transformations were an <u>isometry</u> meaning the transformation preserved size and shape. This means that the pre-image and the image had a 1:1 correspondence and thus were congruent.

II. Dilations

<u>Dilation</u>: A transformation where the figure is either enlarged or reduced in size by a scale factor. The scale factor (k) is the ratio of the corresponding sides.





An enlargement occurs when the scale factor (ratio of corresponding sides) is greater than 1.

A reduction occurs when the scale factor (ratio of corresponding sides) is between 0 and 1.

Notation for a dilation with a scale factor or magnitude of k is $k(x,y)$ or (kx,ky)

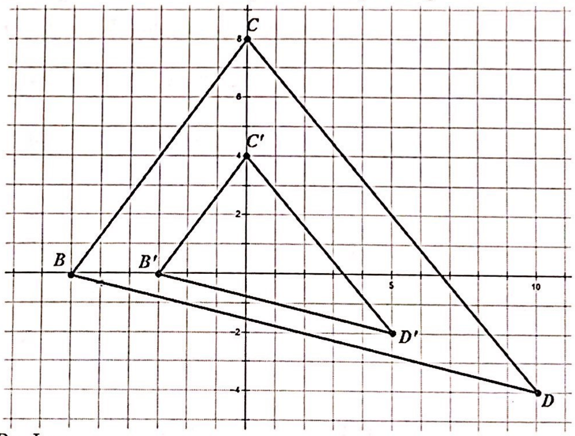
Table Talk: Is a dilation an isometry? Why or why not?

NO ⇒ dilations do not produce figures that are the same size, therefore the figures are not ≅.

Dilations produce figures that are SIMILAR. The figures have the same shape but are not the same size.

Scanned with CamScanner

Examples: List the ordered pairs for the pre-image then list the ordered pairs for the image. Based on these ordered pairs, determine the scale factor for each dilation.



Pre-Image:

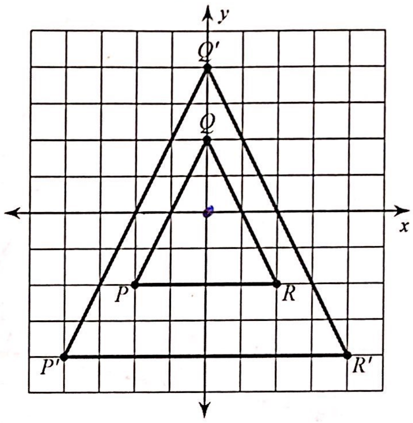
B(−6,0), C(0,8), D(10,−4)

Image:

B'(−3,0), C'(0,4), D'(5,−2)

Scale Factor: 1/2

Function Rule: f(x,y) = (1/2x, 1/2y)



Pre-Image:

P(−2,−2), Q(0,2), R(2,−2)

Image:

P'(−4,−4), Q'(0,4), R'(4,−4)

Scale Factor: 2

Function Rule: f(x,y) = (2x, 2y)

Table Talk: What is the relationship between the ordered pair for the pre-image, the image, and the scale factor?

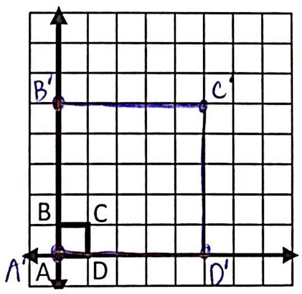
Multiply both the x and y by the scale factor.

1. A dilation has a center of (0,0).
   Find the image of each point for the scale factor given.
   Then identify each as an enlargement or reduction.

   A. $N(-4,-7); k = 0.1$ (−.4, −.7)
   Reduction

   B. $P(-6,2); k = 1.5$ (−9, 3)
   Enlargement

2. Graph $M(x,y) \rightarrow (5x, 5y)$



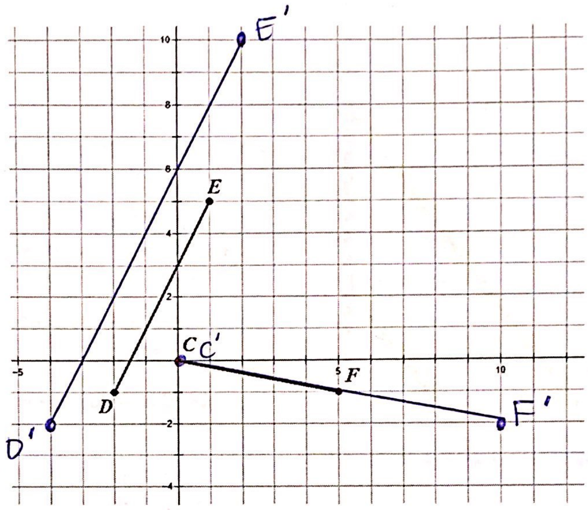
Dilation Investigation #1:

1. Dilate $\overline{DE}$ with center of dilation (0,0) by a scale factor of 2.

D(-2,-1) ⇒ D'(-4,-2)

E(1,5) ⇒ E'(2,10)

$\overline{DE}$ is parallel to $\overline{D'E'}$



2. Dilate $\overline{CF}$ with center of dilation $C(0,0)$ by a scale factor of 2.

C(0,0) ⇒ C'(0,0)

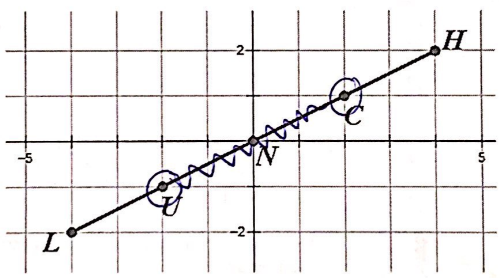
F(5,-1) ⇒ F'(10,-2)

$\overline{CF}$ and $\overline{C'F'}$ lie on the same line segment ⇒ they are coincidental.

You Try: Dilate $\overline{LH}$ by a scale factor of ½ about the origin.

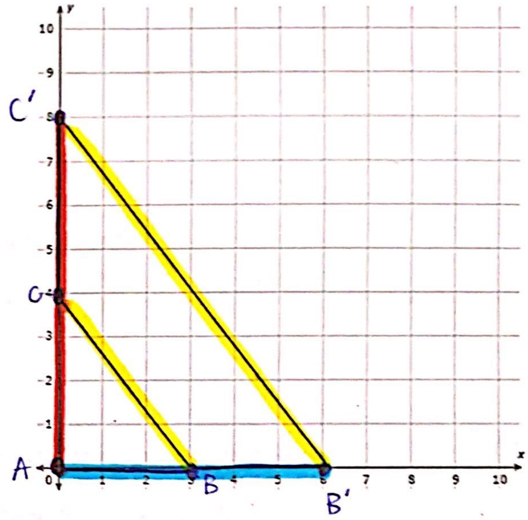
L(-4,-2) ⇒ L'(-2,-1)

H(4,2) ⇒ H'(2,1)



Scanned with CamScanner

Dilation Investigation #2:



1. Graph ΔABC where $A(0,0)$, $B(3,0)$, and $C(0,4)$. Label each point.

2. Dilate ΔABC by a scale factor of 2 centered at the origin.

3. What can you observe about the relationship between the length of the preimage segments and the image segments? The Image segments are twice the length of the pre-image segments.

4. Trace $\overline{AC}$ and $\overline{A'C'}$ in red. Are the corresponding pre-image and image segments parallel or collinear? (same line)

Coincidental/collinear

5. $\overline{AB}$ and $\overline{A'B'}$ in blue. Are the corresponding pre-image and image segments parallel or collinear? (same line)

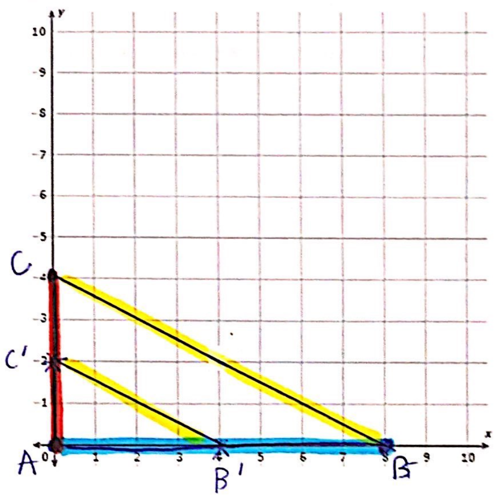
Coincidental/Collinear

6. $\overline{BC}$ and $\overline{B'C'}$ in yellow. Are the corresponding pre-image and image segments parallel or collinear? (same line)

parallel

7. What can you observe about the relationships between the preimage angles and the image angles? You may use patty paper to verify your conjecture. The angle measures stay the same.

You Try:



1. Graph ΔABC where $A(8,4), B(4,2), C(0,4)$. Label each point.

2. Dilate ΔABC by a scale factor of $\frac{1}{2}$ centered at the origin.

3. What can you observe about the relationship between the length of the preimage segments and the image segments? The image segments are ½ the pre-image segments.

4. Trace $\overline{AC}$ and $\overline{A'C'}$ in red. Are the corresponding pre-image and image segments parallel or collinear? (same line)

Coincidental/Collinear

5. $\overline{AB}$ and $\overline{A'B'}$ in blue. Are the corresponding pre-image and image segments parallel or collinear? (same line)

Coincidental/Collinear

6. $\overline{BC}$ and $\overline{B'C'}$ in yellow. Are the corresponding pre-image and image segments parallel or collinear? (same line)

Parallel

7. What can you observe about the relationships between the preimage angles and the image angles? You may use patty paper to verify your conjecture. The angle measures stay the same.

Scanned with CamScanner

Conclusions:

A **dilation** is a transformation that preserves the shape of a geometric figure, but not necessarily the size. It can enlarge (stretch) or reduce (shrink) a figure.

Dilations require:

- **Center of dilation**: a ordered pair about which all points are expanded or contracted.
- **Scale factor/magnitude**: a ratio comparing the **image** (transformed figure) to the **pre-image** (original figure) that determines how much the figure is enlarged or reduced.

Properties of Dilations:

- When a line segment passes through the center of dilation, the line segment and its image lie on the same line. We call these lines collinear/ coincidental.
- When a line segment does not pass through the center of dilation, the line segment and its image are parallel.
- The length of the image of a line segment is equal to the length of the preimage line segment multiplied by the scale factor.
- The distance between the center of a dilation and any point on the image is equal to the scale factor multiplied by the distance between the dilation center and the corresponding point on the pre-image.
- Dilations preserve angle measures.