Unit 6 Lesson 2 – Optimization Problems

Investigation #1: The Horticulture Club at AFHS is building a community garden. They have enough funds to purchase 12 feet of material to build a raised garden.

1. If the Horticulture Club uses the all of their materials, what are some of the possible dimensions for their raised garden?
   Hint: It may be helpful to draw some possible options.

   5 × 1 (5) 4 × 2 (8) 3 × 3 (9) 2 × 4 (8) 1 × 5 (5)

2. Hypothesis: If the Horticulture Club wants garden with the **largest** possible area, what dimensions should they use for the sides?
   Justify your answer. 3 feet by 3 feet ⇒ provides largest area of 9 sq. feet.

3. Write a model for the <u>area</u> of the rectangular pen in terms of the length of one side.

   (3) $6-x$, (3) x, $\frac{12-2x}{2}$, x (3), (3) $6-x$

   $\text{area} = x(6-x)$
   $= 6x - x^2$

4. What kind of function is this (linear, quadratic, exponential)? Explain how you know! Quadratic ⇒ highest exponent is 2.

5. Use technology to graph your model from question #3 above. If the club is looking for the **largest** possible area, which "key feature" of the graph should we look at?

   $y = 6x - x^2$ ⇒ Maximum

6. Use technology the answer to question #5. Does your answer match your hypothesis from question #2 above?

   (3, 9) x = side length, y = max area. When the side lengths are 3 feet the max area will be 9 ft².

Investigation #2: The Horticulture Club at AFHS is building a community garden. They have enough funds to purchase 12 feet of material to build a raised garden. When they build their garden they plan to use the side of the school as one side of their garden.

7. If the Horticulture Club uses the all of their materials, what are some of the possible dimensions for their raised garden?
   Hint: It may be helpful to draw some possible options.

   1 × 10 (10) 2 × 8 (16) 3 × 6 (18) 4 × 4 (16) 5 × 2 (10)

8. Hypothesis: If the Horticulture Club wants garden with the **largest** possible area, what dimensions should they use for the sides?
   Justify your answer. 3 feet by 6 feet ⇒ provides the largest area of 18 ft².

9. Write a model for the <u>area</u> of the rectangular pen in terms of the length of one side.

   (3) x, x (3), $12-2x$ (6)

   $\text{area} = x(12-2x)$
   $= 12x - 2x^2$

10. What kind of function is this (linear, quadratic, exponential)? Explain how you know! Quadratic ⇒ highest exponent is 2.

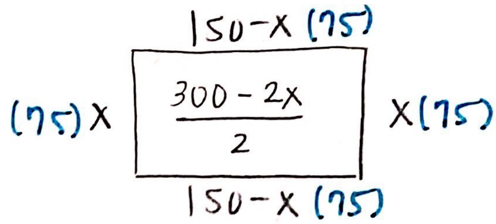
11. Use technology to graph your model from question #3 above. If the club is looking for the **largest** possible area, which "key feature" of the graph should we look at?

    $y = 12x - 2x^2$ ⇒ Maximum

12. Use technology the answer to question #5. Does your answer match your hypothesis from question #2 above?

    (3, 18) x = side length, y = area. When the side length is 3 feet the max area will be 18 ft².

I. Maximizing the Area



1. A farmer has 300 feet of fencing available to make a corral.

A. What is the largest area his corral can be? 5625 ft^2

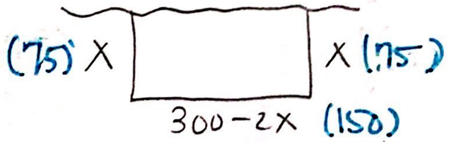
(75, 5625)

B. What dimensions will produce a corral with the **largest** area?

Making the corral 75 feet by 75 feet.

area = $x(150-x)$
$= 150x - x^2$

2. The farmer in question 1 above changed his mind and decided to use the side of his barn as one side of the corral.



A. What is the **largest** area this corral can be? 11,250 ft^2

(75, 11250)

B. What dimensions will produce a corral with the **largest** area?

75 feet by 150 feet.

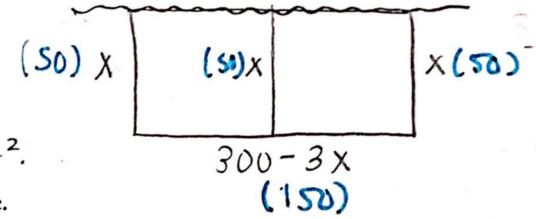
area = $x(300-2x)$
$= 300x - 2x^2$

3. The farmer in the previous two questions realized that he needs to split his one corral into two with a common side. If he still has 300 feet of fencing, now what dimensions will create a **maximum** area?

area = $x(300-3x)$
$= 300x - 3x^2$

(50, 7500)

Making the corral 50 feet by 150 feet for a max area of 7500 ft^2.



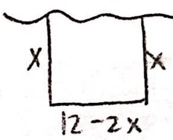
4. Farmer Brown built a rectangular pen for his chickens using 12 meters of fence.
   - He used part of one side of his barn as one **length** of the rectangular pen.
   - He maximized the area using the 12 meters of fence.

Farmer Johnson build a rectangular pen for her chickens using 16 meters of fencing.
   - She used part of one side of her barn as one **length** of the rectangular pen.
   - The length of her pen was 2 meters more than the length of Farmer Brown's pen.
   - The width of her pen was 1 meter more than the width of Farmer Brown's pen.

How much larger is Famer Johnson's rectangular pen than Farmer Brown's?

32 ft^2 − 18 ft^2 = 14 ft^2

FB:



area = $x(12-2x)$
$= 12x - 2x^2$

(3, 18)

3 feet by 6 feet
area of 18 ft^2

FJ:

length = 6+2 = 8
width = 3+1 = 4

8 feet by 4 feet
area of 32 ft^2

III. Optimization of Profit/Cost

1. The Big Brick Bakery sells more bagels when it reduces its prices, but then its profit changes. The function $P(x) = -1000(x - 0.55)^2 + 300$, where x is the price of a bagel in dollars models the bakery's daily profit (in dollars) from selling bagels. The bakery would like to maximize their profits.

A. Can the value of x be negative? Explain. NO ⇒ cannot sell bagel for a negative amt of $.

B. Find the daily profit for selling bagels for $0.40 each.

$P(.40) = -1000(.40-.55)^2 + 300 = \$277.50$

C. What price should the bakery charge to maximize its profit from bagels? Sell for $.55 or 55 cents.

Vertex form ⇒ Max @ (.55, 300)

D. What will their maximum profit be? $300

2. The profit from selling local ballet tickets depends on the ticket price. Using past receipts, it has been found that the profit can be modeled by the function, $p(x) = -15x^2 + 600x + 60$ where x is the price of each ticket. (20, 6060)

A. Find the ticket price that gives the maximum profit. $20

B. Find the maximum profit. $6060