

**I. Projectile Motion** - a form of **motion** experienced by an object (a **projectile**) that is thrown near the Earth's surface and moves along a curved path under the action of gravity only (the effects of air resistance are assumed to be negligible)

$$h(t) = at^2 + bt + c$$

t represents the time the object has been in motion (seconds, minutes, etc)

h(t) represents the height of the object after a specific time (feet, meters)

The coefficients a, b, and c have a special significance.

a is  $\frac{1}{2}$  of the acceleration due to gravity.

- This means our **a** value is always  $-4.9 \text{ m/sec}^2$  or  $-16 \text{ ft/sec}^2$  for projectile motion
- **b** is the initial velocity in  $\text{m/s}$  or  $\text{ft/sec}$  (speed)
- \* **c** is the initial height of the object in  $\text{m}$  or  $\text{ft}$

1. An object thrown into the air is modeled by the equation:  $h(t) = -4.9t^2 + vt + h_0$  where  $v$  is the velocity,  $h_0$  is the initial height above the ground in meters and  $h(t)$  is the height after  $t$  seconds. A person throws a ball with an initial upward velocity of 12  $\text{m/sec}$ . The ball is released when it is 1.8 meters above the ground.

a. Write the equation that would give the height of the ball  $t$  seconds later.

$$h(t) = -4.9t^2 + 12t + 1.8$$

b. After how many seconds will the ball reach its peak?

\* Which key feature is the question asking about? **VERTEX**

$(t, h(t)) \Rightarrow$  TIME is the  $x$ -value.  
 $(1.224, 9.147) \Rightarrow$  after 1.224 seconds the object will reach its peak.

c. What is the **maximum height** of the ball?  $h(t)$  is the  $y$ -value

9.147 meters

d. What is the **y-intercept**?

Describe its significance based on this problem.

$(0, 1.8) \Rightarrow$  The object was released at a height of 1.8 meters above the ground

e. What are the **x-intercepts** of  $h(t)$ ?  $(-1.142, 0)$  and  $(2.591, 0)$

Which zero is relevant? Describe its significance.

$(2.591, 0) \Rightarrow$  it is the amount of time it takes for the object to come back to the ground. It takes 2.591 seconds.

f. At what **time** was the ball at a height of 3 m?

height is the same as  $y$  so graph  $y = 3$

\* determine where the line  $y = 3$  and the parabola intersect

$(0.104, 3)$  and  $(2.345, 3)$

The ball is at a height of 3 meters after .104 seconds and 2.344 sec.  
 (way up) (way down)

On Graphing Calc:

**Find Vertex (Maximum/Minimum)**

On your graph

2<sup>nd</sup> TRACE 4: maximum

move  $x$  using  $\leftarrow$  or  $\rightarrow$

just to left of vertex...ENTER

just to right of vertex...ENTER

on vertex...ENTER

On screen at the bottom

Maximum

$x = \#$

$y = \#$

**Find x-intercept or Intersection**

On your graph

2<sup>nd</sup> TRACE 5: intersect

ENTER on first curve (blue)

ENTER on second curve (red)

move  $x$  to intersection...ENTER

On screen at the bottom

Intersection

$x = \#$

$y = \#$



2. A ball is shot into the air from the top of a building. The height of the ball in feet after  $t$  seconds can be modeled by the equation  $h(t) = -16t^2 + 20t + 50$ .

- a. How tall is the building? 50 feet (y-int).  
 b. How long does it take for the ball to hit the ground? 2.5 sec. (x-int)

what time is the ball at 30 feet? 1.906 sec  
 intersection of  $y_1$  and  $y = 30$

3. During practice, a softball pitcher throws a ball whose height can be modeled by the equation,  $h(t) = -16t^2 + 24t + 1$ , where  $h(t)$  = height in feet and  $t$  = time in seconds.

- a. How long is the ball above a height of 6 feet? 1 sec. (Intersect of  $y_1$  and  $y = 6$ )  
 b. How high is the ball when it is let go from the pitchers hand? 1 foot  
 c. How long was the ball in the air? 1.541 (x-int)  
 d. What was the maximum height of the ball? 10 feet (vertex)  
 e. At what time did the ball reach its maximum height? .75 sec. (vertex)  
 f. What was the initial velocity of the ball? 24 ft/sec.

4. At a fireworks celebration, a bottle rocket is launched upward from the ground with an initial velocity of 160 feet per second. Spectators watch and wonder how high the bottle rocket will go before it begins to descend back toward the ground. The formula for vertical motion of an object is  $h(t) = -16t^2 + vt + s$ , where  $v$  is the initial velocity, and  $s$  is the initial height. Time,  $t$ , is measured in seconds, and height,  $h(t)$ , is measured in feet.

a. Write the quadratic function that describes the height of the bottle rocket after  $t$  seconds  $h(t) = -16t^2 + 160t$

b. How high is the bottle rocket 3 seconds into launch? When is it at this height again?

$$h(3) = -16(3)^2 + 160(3) = 336 \text{ or use table w/ } x=3$$

\* It will be this height again at 7 seconds. ( $y = 336$  or extend table)

Suppose the bottle rocket is launched from the top of a 200-foot-tall building.

$$h(t) = -16t^2 + 160t + 200$$

c. How does this change the maximum height of the rocket? Increased the max height by 200 feet  
 original (5,400) new (5,600)

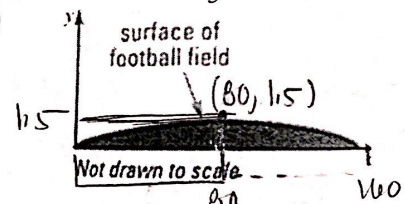
d. How does this change the amount of time it will take the rocket to land on the ground? The rocket will take 1.124 seconds longer to hit the ground.  
 original (10,0) new (11.124,0)

## II. Other Quadratics

5. Although a football field appears to be flat, its surface is actually shaped like a parabola so that water runs off to either side. The cross section of a field with synthetic turf can be modeled by  $y = -.000234(x - 80)^2 + 1.5$  where  $x$  and  $y$  are measured in feet.

a. What is the field's width? 160 feet (Vertex form)  $(80, 1.5)$

b. What is the maximum height of the field's surface? 1.5 feet



6. The woodland jumping mouse can hop surprisingly long distances given its small size. A relatively long hop can be modeled by  $y = -\frac{2}{9}x(x - 6)$  where  $x$  is the horizontal distance in feet and  $y$  is the vertical distance in feet.

(factored form)  $x=0$  and  $x=6$

a. How far can a woodland jumping mouse hop? 6 feet

b. How high can it hop? 2 feet  $(3, 2)$

7. The Golden Gate Bridge in San Francisco has two towers that rise 500 feet above the road and are connected by suspension cables as shown. Each cable forms a parabola with equation  $y = -\frac{1}{8960}(x - 2100)^2 + 8$  where  $x$  and  $y$  are measured in feet.

(vertex form)

a. What is the distance  $d$  between the two towers?

$$100(2) = 4200 \text{ feet}$$

b. What is the height  $l$  above the road of a cable at its lowest point?

8 feet

