

## Unit 5 Day 5: Solve by Quadratic Formula

Solve.  $4x^2 + 12x = -28$

 Fact:  $4x^2 + 12x + 28 = 0$   
 $4(x^2 + 3x + 7) = 0$   
 Cannot be done

SR: Cannot be done because of the  $bx$  term

CS:  $4(x^2 + 3x + \underline{\quad}) - \underline{\quad} = -28$   
 $4(x^2 + 3x + 9/4) - 9/4 = -28$   
 $4(x + 3/2)^2 - 9/4 = -28$   
 $4(x + 3/2)^2 = -19$   
 $\sqrt{(x + 3/2)^2} = \sqrt{-19/4}$   
 $x + 3/2 = \pm i\sqrt{19}/2$

The process of completing the square can be performed on all quadratic equations. If we performed the process on the standard form  $ax^2 + bx + c = 0$ , the solutions for  $x$  would be  $X = \frac{-B \pm \sqrt{B^2 - 4Ac}}{2A}$ .  $x = \frac{-3 \pm i\sqrt{19}}{2}$

This formula is known as the Quadratic Formula. We can determine the values of  $a$ ,  $b$ , and  $c$  and use substitution to find the solutions. This bypasses the process of completing the square and taking the square root.

Videos Here

Use the quadratic formula to solve. Solutions should be written in exact, simplest form.

1. $x^2 + 6x + 7 = 0$ $a=1 \quad b=6 \quad c=7$ $X = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(7)}}{2(1)}$ $X = \frac{-6 \pm \sqrt{8}}{2}$ $X = \frac{-6 \pm 2\sqrt{2}}{2}$ $X = -3 \pm \sqrt{2}$	2. $x^2 + 6x + 8 = 0$ $a=1 \quad b=6 \quad c=8$ $X = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(8)}}{2(1)}$ $X = \frac{-6 \pm \sqrt{4}}{2}$ $X = \frac{-6 \pm 2}{2}$ $X = -4/2 \quad X = -8/2$	3. $x^2 + 6x + 9 = 0$ $a=1 \quad b=6 \quad c=9$ $X = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(9)}}{2(1)}$ $X = \frac{-6 \pm \sqrt{0}}{2}$ $X = \frac{-6 \pm 0}{2}$ $X = -6/2$	4. $x^2 + 6x + 10 = 0$ $a=1 \quad b=6 \quad c=10$ $X = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(10)}}{2(1)}$ $X = \frac{-6 \pm \sqrt{-4}}{2}$ $X = \frac{-6 \pm 2i}{2}$ $X = -3 \pm i$
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$$\{-3 \pm \sqrt{2}\}$$

$$\{-2, -4\}$$

$$\{-3\}$$

$$\{-3 \pm i\}$$

Simplify.

5.  $x = \frac{9 \pm \sqrt{72}}{3}$   
 $X = \frac{9 \pm 6\sqrt{2}}{3}$   
 $X = 3 \pm 2\sqrt{2}$   
 $\{3 \pm 2\sqrt{2}\}$

6.  $x = \frac{10 \pm \sqrt{(-10)^2 - 4(5)(5)}}{2(5)}$   
 $X = \frac{10 \pm \sqrt{0}}{10} = \frac{10 \pm 0}{10}$   
 $X = 1 \quad \{1\}$

7.  $x = \frac{10 \pm \sqrt{(-10)^2 - 4(3)(7)}}{2(3)}$   
 $X = \frac{10 \pm \sqrt{16}}{6} = \frac{10 \pm 4}{6}$   
 $X = \frac{10+4}{6} \quad \frac{10-4}{6}$   
 $X = 14/6 \quad X = 6/6$   
 $\{7/3, 1\}$

Solve using the Quadratic Formula.

8.  $3x^2 - 2x + 5 = 0$   
 $a=3 \quad b=-2 \quad c=5$   
 $X = \frac{-2 \pm \sqrt{(-2)^2 - 4(3)(5)}}{2(3)}$   
 $X = \frac{-2 \pm \sqrt{-56}}{6} = \frac{2 \pm 2i\sqrt{14}}{6}$   
 $X = \frac{1 \pm i\sqrt{14}}{3}$

$\begin{matrix} 56 \\ \diagdown 28 \\ \diagup 14 \\ \diagdown 7 \end{matrix}$

9.  $5x^2 = -6 + 10x$   
 $5x^2 - 10x + 6 = 0$   
 $a=5 \quad b=-10 \quad c=6$   
 $X = \frac{10 \pm \sqrt{(-10)^2 - 4(5)(6)}}{2(5)}$   
 $X = \frac{10 \pm \sqrt{-20}}{10} = \frac{10 \pm 2i\sqrt{5}}{10}$   
 $X = \frac{5 \pm i\sqrt{5}}{5}$

## Discriminant

The value of the expression under the radical symbol is called the Discriminant. The discriminant indicates the number and nature / type of solutions. Review the discriminant in the quadratics that you solved on the front. Generalize your observations.

Description of Discriminant	Examples of Discriminants from #1-9 and the solution(s)	Number & Nature/Type of Solutions	Number of Times Parabola will Cross x-axis
Discriminant is positive and non-perfect square $(b)^2 - 4ac > 0$ and is not perfect square	#1, #5 ↓ ↓ 8 72 $(-3 \pm \sqrt{2}) (3 \pm 2\sqrt{2})$	2 Real Irrational Solutions	2
Discriminant is positive and perfect square $(b)^2 - 4ac > 0$ and is perfect square	#2, #7 ↓ ↓ 4 16 $\{-2, -4\} \{7/3, 13\}$	2 Real Rational Solutions	2
Discriminant is 0 $(b)^2 - 4ac = 0$	#3, #6 ↓ ↓ $\{3^0\} \{0\}$	1 Real	1
Discriminant is negative $(b)^2 - 4ac < 0$	#4, #8, #9 ↓ ↓ ↓ -4 -56 -20 $\{-3 \pm i\} \left\{ \frac{1 \pm i\sqrt{14}}{3} \right\} \left\{ \frac{5 \pm i\sqrt{5}}{5} \right\}$	2 Complex/imaginary Solutions	0

## Choosing the Best Solving Method

Method	Form	Most EFFICIENT to use when...
Factoring	$ax^2 + bx + c = 0$ $( )( ) = 0$	<ul style="list-style-type: none"> <li>if <math>a = 1</math></li> <li>if <math>c = 0</math></li> <li>if <math>a</math> and <math>c</math> are prime</li> <li>If <math>b = 0 \Rightarrow</math> difference of squares?</li> </ul>
Square Root	$ax^2 + c = 0$ $( )^2 = \#$	<ul style="list-style-type: none"> <li>if <math>b = 0</math></li> <li>adjusted perfect square (vertex form)</li> </ul>
Complete the Square	Any... Rewrite left side in vertex form	<ul style="list-style-type: none"> <li>if <math>a = 1</math> and <math>b</math> is <u>EVEN</u></li> </ul>
Quadratic Formula	Any... $ax^2 + bx + c = 0$	<ul style="list-style-type: none"> <li>if <math>b^2 - 4ac</math> is negative</li> <li>if nothing else is easier</li> </ul>