# Unit 5 Day 3: Solving by Completing the Square

Solve by Factoring or Square Root: $x^2 + 8x - 7 = 0$

↳ since there is a Bx term we cannot √.

Cannot be factored ⇒ no factors of −7 that add to +8.

Not all quadratic equations can be solved directly by factoring or by taking the square root of both sides.

However, the square root method is useful because <u>any</u> quadratic equation can be written in the form $(x - h)^2 = k$.

Recall: The process of changing an equation that is in standard form to vertex form is called Complete the Square.

**Table Talk:** We previously discovered the relationship between the middle and last terms of any perfect square. What was it?

$(B/2)^2 = C$

Based on what you wrote for the table talk section above, fill in the blank to make each quadratic a perfect square.

$y^2 + 14y + \underline{49}$ $(14/2)^2$ $\quad a^2 - 10a + \underline{25}$ $(-10/2)^2$ $\quad m^2 + 7m + \underline{49/4}$ $(7/2)^2$

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We are now ready to use our observations to solve $x^2 + 8x - 7 = 0$

Step 1: Rewrite the left side of the equation in vertex form by completing the square. Leave the right side of the equation alone!

$(8/2)^2$

$x^2 + 8x + \_\_ - 7 - \_\_ = 0$

$x^2 + 8x + 16 - 7 - 16 = 0$

$(x+4)^2 - 23 = 0$

Step 2: Isolate the $(x - h)^2$.

$(x+4)^2 - 23 = 0$

$+23 \quad +23$

$(x+4)^2 = 23$

Step 3: Take the square root of both sides. Don't forget to include the ± when you do this.

$\sqrt{(x+4)^2} = \sqrt{23}$

$x + 4 = \pm\sqrt{23}$

Step 4: Solve for the variable.

$x + 4 = \pm\sqrt{23}$

$x + 4 = \sqrt{23}$ $\quad x + 4 = -\sqrt{23}$

$-4 \quad -4$ $\quad -4 \quad -4$

$x = -4 + \sqrt{23}$ $\quad x = -4 - \sqrt{23}$

$\{-4 \pm \sqrt{23}\}$

Examples:

1. $x^2 - 6x - 4 = -2$

$(-6/2)^2$

$x^2 - 6x + \_\_ - 4 - \_\_ = -2$

$x^2 - 6x + 9 - 4 - 9 = -2$

$(x-3)^2 - 13 = -2$

$(x-3)^2 = 11$

$\sqrt{(x-3)^2} = \sqrt{11}$

$x - 3 = \pm\sqrt{11}$

$x = 3 \pm \sqrt{11}$ $\{3 \pm \sqrt{11}\}$

2. $b^2 - 6b + 82 = 5$

$(-6/2)^2$

$b^2 - 6b + \_\_ + 82 - \_\_ = 5$

$b^2 - 6b + 9 + 82 - 9 = 5$

$(b-3)^2 + 73 = 5$

$(b-3)^2 = -68$

$\sqrt{(b-3)^2} = \sqrt{-68}$

$b - 3 = \pm\sqrt{-1}\sqrt{68}$

$\sqrt{68}$: $68 = 2 \cdot 34$, $34 = 2 \cdot 17$

$b - 3 = \pm 2i\sqrt{17}$

$b = 3 \pm 2i\sqrt{17}$ $\{3 \pm 2i\sqrt{17}\}$

3. $x^2 + 4x + 13 = 0$ $(4/2)^2$

$x^2 + 4x + \_\_ + 13 - \_\_ = 0$

$x^2 + 4x + 4 + 13 - 4 = 0$

$(x+2)^2 + 9 = 0$

$(x+2)^2 = -9$

$\sqrt{(x+2)^2} = \sqrt{-9}$

$x + 2 = \pm\sqrt{-1}\sqrt{9}$

$x + 2 = \pm 3i$

$x = -2 \pm 3i$ $\{-2 \pm 3i\}$

4. $m^2 - 4m - 50 = -9$

$m^2 - 4m + \_\_ - 50 - \_\_ = -9$

$m^2 - 4m + 4 - 50 - 4 = -9$

$(m-2)^2 - 54 = -9$

$(m-2)^2 = 45$

$\sqrt{(m-2)^2} = \sqrt{45}$

$m - 2 = \pm\sqrt{45}$

$\sqrt{45}$: $45 = 9 \cdot 5$, $9 = 3 \cdot 3$

$m - 2 = \pm 3\sqrt{5}$

$m = 2 \pm 3\sqrt{5}$ $\{2 \pm 3\sqrt{5}\}$

5. $2x^2 - 8x + 3 = 195$

$2(x^2 - 4x + \_\_) + 3 - \_\_ = 195$

$2(x^2 - 4x + 4) + 3 - 8 = 195$

$2(x-2)^2 - 5 = 195$

$2(x-2)^2 = 200$

$(x-2)^2 = 100$

$\sqrt{(x-2)^2} = \sqrt{100}$

$x - 2 = \pm 10$

$x - 2 = 10$ $\quad x - 2 = -10$

$x = 12$ $\quad x = -8$ $\{12, -8\}$

YOU TRY: $x^2 + 10x + 40 = -1$

$x^2 + 10x + \_\_ + 40 - \_\_ = -1$

$x^2 + 10x + 25 + 40 - 25 = -1$

$(x+5)^2 - 15 = -1$

$(x+5)^2 = 14$

$\sqrt{(x+5)^2} = \sqrt{14}$

$x + 5 = \pm\sqrt{14}$

$x = -5 \pm \sqrt{14}$ $\{-5 \pm \sqrt{14}\}$

6. $4x^2 + 12x = -28$

$4(x^2 + 3x + \_\_) - \_\_ = -28$ $\quad 9/4$

$4(x^2 + 3x + 9/4) - 9 = -28$

$4(x + 3/2)^2 - 9 = -28$

$4(x + 3/2)^2 = -19$

$(x + 3/2)^2 = \frac{-19}{4}$

$\sqrt{(x+3/2)^2} = \sqrt{-19/4}$

$x + 3/2 = \pm\sqrt{-1} \cdot \frac{\sqrt{19}}{\sqrt{4}}$

$x + 3/2 = \pm \frac{i\sqrt{19}}{2}$

$x = -3/2 \pm i\sqrt{19}/2$ $\left\{-\frac{3 \pm i\sqrt{19}}{2}\right\}$