Unit 5 Lesson 2 - Solving Quadratics by Square Rooting both Sides

Recall that when we solve a quadratic we are using algebra to find the X-intercepts or the value(s) of the variable that satisfy the equation.

We previously learned that one method we can use to do this is to solve using factoring.

Solve: $y = 2x^2 - 9$

Solve: $y = 2x^2 + 9$

Unfortunately, not all equations can be solved by factoring and not all parabolas will cross the x-axis.
This is why we need additional methods to solve for the x-intercepts or the value of x that satisfies the equation.

For equations that have a ***b*** value of zero or contain a perfect square with adjustments, we can solve for *x* algebraically by **isolating** x^2 ***or*** $(x-h)^2$ and then taking the square root of both sides of the equation.

Whenever YOU take the square root of a number, you must be sure to include the $\pm$ symbol to represent both the positive and negative roots and to ensure that you get two solutions.

Examples: Solve each quadratic by taking the square root of both sides.

1. $\sqrt{x^2} = \sqrt{121}$

$x = \pm 11$

$\{11, -11\}$

2. $\frac{3x^2}{3} = \frac{108}{3}$

$\sqrt{x^2} = \sqrt{36}$

$x = \pm 6$

$\{6, -6\}$

3. $2x^2 + 5 = 133$

$-5 \quad -5$

$\frac{2x^2}{2} = \frac{128}{2}$

$\sqrt{x^2} = \sqrt{64}$

$x = \pm 8$

$\{8, -8\}$

4. $-3x^2 - 5 = -80$

$+5 \quad +5$

$\frac{-3x^2}{-3} = \frac{-75}{-3}$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

$\{5, -5\}$

5. $\frac{1}{2}x^2 = 8$

$\frac{}{1/2} \quad \frac{}{1/2}$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm 4$

$\{4, -4\}$

6. $\sqrt{(x-5)^2} = \sqrt{49}$

$x - 5 = \pm 7$

$+5 \quad +5$

$x = 5 \pm 7$

$5+7 \qquad 5-7$

$12 \qquad -2$

$\{12, -2\}$

7. $\frac{2(x+1)^2}{2} = \frac{8}{2}$

$\sqrt{(x+1)^2} = \sqrt{4}$

$x + 1 = \pm 2$

$-1 \quad -1$

$x = -1 \pm 2$

$-1+2 \qquad -1-2$

$1 \qquad -3$

$\{1, -3\}$

8. $2(a-6)^2 - 45 = 53$

$+45 \quad +45$

$\frac{2(a-6)^2}{2} = \frac{98}{2}$

$\sqrt{(a-6)^2} = \sqrt{36}$

$a - 6 = \pm 6$

$+6 \quad +6$

$a = 6 \pm 6$

$6+6 \qquad 6-6$

$12 \qquad 0$

$\{12, 0\}$

9. $\frac{-2(x+2)^2}{-2} = \frac{-18}{-2}$

$\sqrt{(x+2)^2} = \sqrt{9}$

$x + 2 = \pm 3$

$-2 \quad -2$

$x = -2 \pm 3$

$-2+3 \qquad -2-3$

$1 \qquad -5$

$\{1, -5\}$

Sometimes when we take the square root, the number we take the square root of is NOT a perfect square.
We should simplify the radical to find the EXACT value of our answer.

Examples where radicand (value under the radical) is not a perfect square.

1. $x = \pm\sqrt{24} = \sqrt{2\cdot2\cdot2\cdot3}$
   2 12
   2 6
   2 3
   $= \pm 2\sqrt{6}$

2. $x = \pm\sqrt{18} = \sqrt{2\cdot3\cdot3}$
   2 9
   3 3
   $= \pm 3\sqrt{2}$

3. $x = \pm\sqrt{27} = \sqrt{3\cdot3\cdot3}$
   3 9
   3 3
   $= \pm 3\sqrt{3}$

Sometimes when we take the square root, the number we take the square root of is NOT positive.
A negative value under the radical indicates that the quadratic equation does not have real roots. The parabola will never cross the x-axis or have real x-intercepts.

We can still find a value of x that satisfies the equation. These values are a part of the complex number system and are called Imaginary numbers. To indicate the solution is imaginary, we use i. $i = \sqrt{-1}$

Examples where radicand (value under the radical) is not positive.

4. $x = \pm\sqrt{-16}$
   $\sqrt{-1}\ \sqrt{16}$
   $\{\pm 4i\}$

5. $x = \pm\sqrt{-18} = \sqrt{-1}\cdot\sqrt{3\cdot3\cdot2}$
   $\sqrt{-1}\ \sqrt{18}$
   9 2
   3 3
   $= \pm 3i\sqrt{2}$

6. $x = \pm\sqrt{-27} = \sqrt{-1}\cdot\sqrt{3\cdot3\cdot3}$
   $\sqrt{-1}\ \sqrt{27}$
   $= 3i\sqrt{3}$

YOU TRY:

7. $x = \pm\sqrt{8} = \sqrt{2\cdot2\cdot2}$
   2 4
   2 2
   $= \pm 2\sqrt{2}$

8. $x = \pm\sqrt{-32} = \sqrt{-1}\cdot\sqrt{2\cdot2\cdot2\cdot2\cdot2}$
   $\sqrt{-1}\ \sqrt{32}$
   2 16
   2 8
   2 4
   2 2
   $= \pm 4i\sqrt{2}$

9. $x = \pm\sqrt{12} = \pm\sqrt{2\cdot2\cdot3}$
   2 6
   2 3
   $= \pm 2\sqrt{3}$

Examples: Solve each quadratic by taking the square root of both sides. Simplify to get an EXACT answer.

1. $\sqrt{x^2} = \sqrt{75}$ $\{\pm 5\sqrt{3}\}$
   $x = \pm\sqrt{75}$
   3 25
   5 5
   $= \pm\sqrt{3\cdot5\cdot5}$
   $= \pm 5\sqrt{3}$

2. $\frac{3x^2}{3} = \frac{180}{3}$ $\{\pm 2\sqrt{15}\}$
   $\sqrt{x^2} = \sqrt{60}$
   $x = \pm\sqrt{60} = \pm\sqrt{2\cdot2\cdot5\cdot3}$
   2 30
   2 15
   5 3
   $= \pm 2\sqrt{15}$

3. $\sqrt{(x+2)^2} = \sqrt{-12}$ $\{-2 \pm 2i\sqrt{3}\}$
   $(x+2) = \pm\sqrt{-1}\cdot\sqrt{12}$
   $(x+2) = \pm\sqrt{-1}\cdot\sqrt{2\cdot2\cdot3}$
   $(x+2) = \pm 2i\sqrt{3}$
   $-2 \quad -2$
   $x = -2 \pm 2i\sqrt{3}$

4. $2(x+3)^2 + 2 = -8$
   $-2 \quad -2$
   $\frac{2(x+3)^2}{2} = \frac{-10}{2}$
   $\sqrt{(x+3)^2} = \sqrt{-5}$
   $x+3 = \pm\sqrt{-1}\sqrt{5}$
   $x+3 = \pm i\sqrt{5}$
   $-3 \quad -3$
   $x = -3 \pm i\sqrt{5}$
   $\{-3 \pm i\sqrt{5}\}$

5. $\sqrt{(3x)^2} = \sqrt{72}$ $\{\pm 2\sqrt{2}\}$
   $3x = \sqrt{72}$
   2 36
   2 18
   2 9
   3 3
   $3x = \sqrt{2\cdot2\cdot2\cdot3\cdot3}$
   $\frac{3x}{3} = \pm\frac{6\sqrt{2}}{3}$ $x = \pm 2\sqrt{2}$

6. $\sqrt{(16x-4)^2} = \sqrt{128}$ $\left\{\frac{1 \pm 2\sqrt{2}}{4}\right\}$
   $(16x-4) = \sqrt{128}$
   2 64
   8 8
   $(16x-4) = \pm 8\sqrt{2}$
   $+4 \quad +4$
   $\frac{16x}{16} = \frac{4 \pm 8\sqrt{2}}{16}$
   $x = \frac{4 \pm 8\sqrt{2}}{16} = \frac{1 \pm 2\sqrt{2}}{4}$