Unit 5 Lesson 1 - Solving Quadratics by Factoring (finding x-intercepts)

To **SOLVE** means to find the values of the variable that will make the equation true.

W... we solve a quadratic we are trying to find the x-intercepts or the place where the parabola crosses the x-axis.

To find the solutions, roots, or zeros of a quadratic function (given as $y =$ or $f(x) =$), substitute 0 for y / f(x) since x-intercepts have a y-value of 0.

We have a total of 4 methods we will learn to solve quadratics – to help us find those x-intercepts.

1. Factoring
2. Square Root
3. Complete the Square
4. Quadratic Formula

One key to solving by factoring is the Zero Product Property:

*Have students create a problem with product of 0. Share to help them see one factor MUST be 0.

if a · b = 0 then a = 0 or b = 0

Examples – Apply the Zero Product Property to *verify each of these on a graph*

1. $(x+3)(x-2) = 0$ ("a" "b")
   $x+3=0 \quad x-2=0$
   $x=-3 \quad x=2$
   $(-3,0) \quad (2,0)$

2. $(2x-5)(x-3) = 0$ ("a" "b")
   $2x-5=0 \quad x-3=0$
   $2x=5 \quad x=3$
   $x=5/2$
   $(5/2,0) \quad (3,0)$

3. $x(x+3) = 0$ ("a" "b")
   $x=0 \quad x+3=0$
   $x=-3$
   $(0,0) \quad (-3,0)$

4. $0 = 2x(x-9)$ ("a" "b")
   $2x=0 \quad x-9=0$
   $x=0 \quad x=9$
   $(0,0) \quad (9,0)$

Solve $y = x^2 + 5x + 6$

Steps:

1. Put the equation in Standard Form - $ax^2 + bx + c = 0$

   $0 = x^2+5x+6 \Rightarrow x^2+5x+6=0$

2. Factor the quadratic

   $(x+3)(x+2)=0$

3. Use the Zero Product Property (to set each factor equal to 0):

   $(x+3)=0 \quad (x+2)=0$

4. ...te the variable.

   $x+3=0 \quad x+2=0$
   $x=-3 \quad x=-2$

5. Write a solution set

   $\{-3,-2\}$

Examples:

1. $f(x) = x^2 + 8x + 7$

$0 = x^2+8x+7$
$= (x+7)(x+1)$
$(x+7)=0 \quad (x+1)=0$
$-7 \; -7 \quad -1 \; -1$
$x=-7 \quad x=-1$
$\{-7,-1\}$

*Means the parabola will cross the x-axis at (-7,0) and (-1,0)

2. $x^2 = -x + 12$

$x^2+x-12=0$
$(x+4)(x-3)=0$
$(x+4)=0 \quad x-3=0$
$-4 \; -4 \quad +3 \; +3$
$x=-4 \quad x=3$
$\{-4,3\}$

*Means the parabola will cross the x-axis at (-4,0) and (3,0).

3. $y = 8x^2 + 2x - 3$

$0 = 8x^2+2x-3$
$0=(4x+3)(2x-1)$
$4x+3=0 \quad 2x-1=0$
$4x=-3 \quad 2x=1$
$x=-3/4 \quad x=1/2$
$\{-3/4, 1/2\}$

*Means the parabola will cross the x-axis at (-3/4,0) and (1/2,0)

4. $2x^2 - 24x = -72$

$2x^2-24x+72=0$
$2(x^2-12x+36)=0$
$2(x-6)(x-6)=0$
$x-6=0 \quad x-6=0$
$x=6 \quad x=6$
$\{6\}$

*Means the parabola will cross the x-axis <u>ONCE</u> at (6,0).

5. $x^2 + 2x - 35 = 0$

$(x+7)(x-5)=0$
$x+7=0 \quad x-5=0$
$-7 \; -7 \quad +5 \; +5$
$x=-7 \quad x=5$
$\{-7,5\}$

6. $f(x) = 4x^2 - 9x + 2$

$0=4x^2-9x+2$
$0=(4x-1)(x-2)$
$4x-1=0 \quad x-2=0$
$4x=1 \quad x=2$
$x=1/4$
$\{1/4, 2\}$

7. $6a^2 + 36a = y$

$6a^2+36a=0$
$6a(a+6)=0$
$\frac{6a}{6}=\frac{0}{6} \quad a+6=0$
$-6 \; -6$
$a=0 \quad a=-6$
$\{0,-6\}$

8. $x^2 + 30 = 7x$

$x^2-7x+30=0$
$(x-3)(x-10)=0$
$x-3=0 \quad x-10=0$
$x=3 \quad x=10$
$\{3,10\}$

Part 2: Writing a quadratic equation given the x-intercepts or solutions.

Solve each of the following quadratic equations.

$f(x) = 2(x+3)(x-1)$ $g(x) = -3(x+3)(x-1)$ $h(x) = 0.5(x+3)(x-1)$

Compare and contrast the equations for $f(x), g(x)$, and $h(x)$ as well as the solutions for the equations.

Many quadratic equations can have the same x-intercepts if the linear factors are the same, but the a values are different. When we are writing a quadratic equation from the solutions, we need to go through the process of solving for the solutions in reverse order. Additionally, we need to determine the a value that will force the parabola to pass through a specific point on the graph.

Solve $y = 2x^2 + 20x + 32$	Write an equation in factored form given the solutions $x = -8$ and $x = -2$ that passes through $(-3, -10)$
$2(x^2 + 10x + 16) = 0$	$x + 8 = 0$ and $x + 2 = 0$ (Write the linear factors by rearranging each equation to equal zero.)
$2(x+8)(x+2) = 0$	$y = a(x+8)(x+2)$ (Multiply the linear factors and place a as a possible GCF.
$x + 8 = 0$ and $x + 2 = 0$	$-10 = a(-3+8)(-3+2)$ Replace x and y with the values from the ordered pair.
$x = -8$ and $x = -2$	$-10 = a(5)(-1)$ Simplify
	$-10 = -5a$ Simplify
	$2 = a$ Solve for a.
	$y = 2(x+8)(x+2)$ This is the equation in FACTORED FORM.

Examples: Write a quadratic equation in factored form given the following x-intercepts and points on the parabolas.

1. $\{-4, 5\}$ and passes through $(1, 40)$

$x = -4$ $x = 5$
$x+4=0$ $x-5=0$
$y = a(x+4)(x-5)$
$40 = a(1+4)(1-5)$
$40 = a(5)(-4)$
$40 = -20a$
$a = -2$
$y = -2(x+4)(x-5)$ factored form
$y = -2x^2+2x+40$ standard form

2. $\{\frac{1}{3}, -2\}$ and passes through $(0,2)$

$x = 1/3$ $x = -2$
$3x = 1$ $x+2=0$
$3x-1=0$
$y = a(x+2)(3x-1)$
$2 = a(0+2)(0-1)$
$2 = a(2)(-1)$
$2 = -2a$
$a = -1$
$y = -(x+2)(3x-1)$ factored form
$y = -3x^2-5x+2$ standard form

3. $\{\frac{4}{3}, 1\}$ and passes through $(-1, -28)$

$x = 4/3$ $x = 1$
$3x = 4$ $x-1=0$
$3x-4=0$
$y = a(3x-4)(x-1)$
$-28 = a(-7)(-2)$
$-28 = 14a$
$a = -2$
$y = -2(3x-4)(x-1)$ factored form
$y = -6x^2+14x-8$

4. $\{0, 3\}$ and passes through $(1, -1)$

$x = 0$ $x = 3$
$x-3=0$
$y = ax(x-3)$
$-1 = a(1)(1-3)$
$-1 = -2a$
$a = 1/2$
$y = 1/2(x)(x-3)$ factored form
$y = 1/2x^2 - 3/2x$ standard form

5. $\{-8\}$ and passes through $(2, 100)$

$x = -8$ $x = -8$
$x+8=0$ $x+8=0$
$a(x+8)(x+8) = y$
$100 = a(10)(-6)$
$100 = -60a$
$a = -60/100 = -6/10 = -3/5$
$y = -3/5(x+8)(x+8)$ factored form
$y = -3/5x^2 + -48/5x - 192/5$ standard form